Session 2   **Mathematics Section**

Multiple-Choice Questions

1. What is the value of $\sqrt[3]{5} \times 25^{\frac{1}{3}}$? [2 points]

① 1   ② 2   ③ 3   ④ 4   ⑤ 5

2. For the function $f(x) = x^3 - 8x + 7$, what is the value of $\lim_{h \to 0} \dfrac{f(2+h) - f(2)}{h}$? [2 points]

① 1   ② 2   ③ 3   ④ 4   ⑤ 5

3. Let $\{a_n\}$ be a geometric sequence where both the first term and the common ratio are equal to a positive constant k. If

$$\frac{a_4}{a_2} + \frac{a_2}{a_1} = 30,$$

what is the value of k? [3 points]

① 1   ② 2   ③ 3   ④ 4   ⑤ 5

4. If the function

$$f(x) = \begin{cases} 5x + a & (x < -2) \\ x^2 - a & (x \geq -2) \end{cases}$$

is continuous on the set of all real numbers, what is the value of the constant a? [3 points]

① 6   ② 7   ③ 8   ④ 9   ⑤ 10

5. For the function $f(x) = (x^2 + 1)(3x^2 - x)$, what is the value of $f'(1)$? [3 points]

① 8 　　② 10 　　③ 12 　　④ 14 　　⑤ 16

7. If the polynomial function $f(x)$ satisfies

$$\int_0^x f(t)\, dt = 3x^3 + 2x$$

for all real numbers x, what is the value of $f(1)$?

[3 points]

① 7 　　② 9 　　③ 11 　　④ 13 　　⑤ 15

6. If $\cos\left(\dfrac{\pi}{2} + \theta\right) = -\dfrac{1}{5}$, what is the value of $\dfrac{\sin\theta}{1 - \cos^2\theta}$?

[3 points]

① -5 　　② $-\sqrt{5}$ 　　③ 0 　　④ $\sqrt{5}$ 　　⑤ 5

8. For two real numbers $a = 2\log\dfrac{1}{\sqrt{10}} + \log_2 20$ and $b = \log 2$, what is the value of $a \times b$? [3 points]

① 1   ② 2   ③ 3   ④ 4   ⑤ 5

10. For the function $f(x) = a\cos bx + 3$ defined on the closed interval $[0, 2\pi]$, let (a, b) be an ordered pair of natural numbers such that $f(x)$ has a maximum value of 13 at $x = \dfrac{\pi}{3}$. What is the minimum value of $a + b$? [4 points]

① 12   ② 14   ③ 16   ④ 18   ⑤ 20

9. For the function $f(x) = 3x^2 - 16x - 20$, if

$$\int_{-2}^{a} f(x)\,dx = \int_{-2}^{0} f(x)\,dx,$$

what is the value of the positive constant a? [4 points]

① 16   ② 14   ③ 12   ④ 10   ⑤ 8

11. A point P starts moving at time $t = 0$ along a number line, and its position x at time t $(t \geq 0)$ is given by

$$x = t^3 - \frac{3}{2}t^2 - 6t$$

What is the acceleration of point P at the moment its direction of motion changes after starting? [4 points]

① 6   ② 9   ③ 12   ④ 15   ⑤ 18

12. Let $\{a_n\}$ be a sequence with $a_1 = 2$ and $\{b_n\}$ be an arithmetic sequence with $b_1 = 2$. If they satisfy

$$\sum_{k=1}^{n} \frac{a_k}{b_{k+1}} = \frac{1}{2}n^2$$

for all natural numbers n, what is the value of $\sum_{k=1}^{5} a_k$?
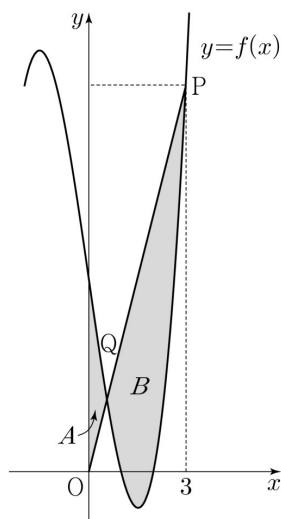
[4 points]

① 120   ② 125   ③ 130   ④ 135   ⑤ 140

13. Let $f(x)$ be a cubic function with a leading coefficient of 1 satisfying
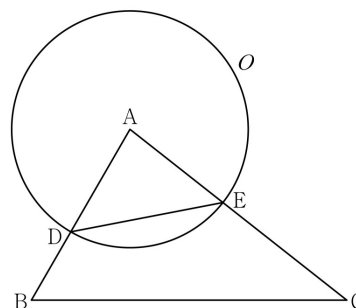
$$f(1)=f(2)=0,\ f'(0)=-7.$$

For the origin O and point P$(3, f(3))$, let Q be the point other than P where segment OP intersects the curve $y=f(x)$. Let A be the area of the region bounded by the curve $y=f(x)$, the y-axis, and segment OQ, and let B be the area of the region bounded by the curve $y=f(x)$ and segment PQ. What is the value of $B-A$? [4 points]

① $\dfrac{37}{4}$　② $\dfrac{39}{4}$　③ $\dfrac{41}{4}$　④ $\dfrac{43}{4}$　⑤ $\dfrac{45}{4}$

14. As shown in the figure, in triangle ABC, let D be a point on segment AB such that $\overline{AD} : \overline{DB}=3:2$. Let O be a circle centered at A passing through D, and let E be the point where circle O intersects segment AC.
If $\sin A : \sin C = 8:5$ and the ratio of the area of triangle ADE to the area of triangle ABC is $9:35$. Given that the radius of the circumcircle of triangle ABC is 7, what is the maximum value of the area of triangle PBC for a point P on circle O? (Note: $\overline{AB}<\overline{AC}$) [4 points]

① $18+15\sqrt{3}$　② $24+20\sqrt{3}$　③ $30+25\sqrt{3}$

④ $36+30\sqrt{3}$　⑤ $42+35\sqrt{3}$

15. For a constant a $\left(a \neq 3\sqrt{5}\right)$ and a quadratic function $f(x)$ with a negative leading coefficient, let the function $g(x)$ be defined as:

$$g(x) = \begin{cases} x^3 + ax^2 + 15x + 7 & (x \leq 0) \\ f(x) & (x > 0) \end{cases}$$

If $g(x)$ satisfies the following conditions:

> (a) The function $g(x)$ is differentiable on the set of all real numbers.
>
> (b) The number of distinct real roots of the equation $g'(x) \times g'(x-4) = 0$ is 4.

What is the value of $g(-2) + g(2)$? [4 points]

① 30   ② 32   ③ 34   ④ 36   ⑤ 38

Short-Answer Questions

16. What is the value of the real number x satisfying the equation $\log_2(x-3) = \log_4(3x-5)$? [3 points]

17. For a polynomial function $f(x)$, if $f'(x) = 9x^2 + 4x$ and $f(1) = 6$, what is the value of $f(2)$? [3 points]

18. Let $\{a_n\}$ be a sequence satisfying

$$a_n + a_{n+4} = 12$$

for all natural numbers n, What is the value of $\displaystyle\sum_{n=1}^{16} a_n$?

[3 points]

19. For a positive constant a, let the function $f(x)$ be defined as

$$f(x) = 2x^3 - 3ax^2 - 12a^2 x.$$

If the local maximum value of $f(x)$ is $\dfrac{7}{27}$, what is the value of $f(3)$? [3 points]

20. Let k be the x-coordinate of the intersection point of the curve $y = \left(\dfrac{1}{5}\right)^{x-3}$ and the line $y = x$. A function $f(x)$ defined on the set of all real numbers satisfies the following conditions:

$$f(x) = \left(\dfrac{1}{5}\right)^{x-3} \text{ and } f(f(x)) = 3x \text{ for all real numbers } x > k.$$

What is the value of $f\left(\dfrac{1}{k^3 \times 5^{3k}}\right)$? [4 points]

21. For integers a and b such that funciton

$f(x) = x^3 + ax^2 + bx + 4$ satisfies the following condition, what is the maximum value of $f(1)$? [4 points]

> For all real numbers α, the limit $\displaystyle\lim_{x \to \alpha} \frac{f(2x+1)}{f(x)}$ exists.

22. For all sequences $\{a_n\}$ with integer terms that satisfy the following conditions, what is the sum of all possible values of $|a_1|$? [4 points]

> (a) For all natural numbers n,
> $$a_{n+1} = \begin{cases} a_n - 3 & (\text{if } |a_n| \text{ is odd}) \\ \dfrac{1}{2}a_n & (\text{if } a_n = 0 \text{ or } |a_n| \text{ is even}) \end{cases}$$
>
> (b) The minimum natural number m satisfying
> $|a_m| = |a_{m+2}|$ is 3.

2025 College Scholastic Ability Test

Mathematics Section (Calculus)

Multiple-Choice Questions

23. What is the value of $\lim\limits_{x \to 0} \dfrac{3x^2}{\sin^2 x}$? [2 points]

① 1   ② 2   ③ 3   ④ 4   ⑤ 5
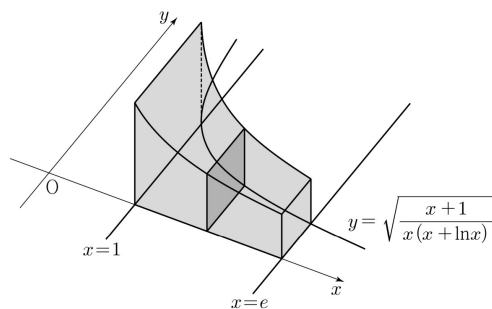
24. What is the value of $\displaystyle\int_0^{10} \dfrac{x+2}{x+1}\,dx$? [3 points]

① $10+\ln 5$ ② $10+\ln 7$ ③ $10+2\ln 3$

④ $10+\ln 11$ ⑤ $10+\ln 13$

25. For a sequence $\{a_n\}$, if $\lim\limits_{n\to\infty}\dfrac{na_n}{n^2+3}=1$, what is the value

of $\lim\limits_{n\to\infty}\left(\sqrt{a_n^{\,2}+n}-a_n\right)$? [3 points]

① $\dfrac{1}{3}$ ② $\dfrac{1}{2}$ ③ 1   ④ 2   ⑤ 3

26. As shown in the figure, there is a solid whose base is the region bounded by the curve $y=\sqrt{\dfrac{x+1}{x(x+\ln x)}}$, the x-axis, and the lines $x=1$ and $x=e$. If every cross-section of this solid perpendicular to the x-axis is a square, what is the volume of this solid? [3 points]

① $\ln(e+1)$ ② $\ln(e+2)$ ③ $\ln(e+3)$

④ $\ln(2e+1)$ ⑤ $\ln(2e+2)$

27. For a cubic function $f(x)$ with a leading coefficient of 1, let the function $g(x)$ be defined as

$$g(x) = f(e^x) + e^x.$$

If the tangent line to the curve $y = g(x)$ at the point $(0, g(0))$ is the x-axis and the function $g(x)$ has an inverse function $h(x)$, what is the value of $h'(8)$? [3 points]

① $\dfrac{1}{36}$　　② $\dfrac{1}{18}$　　③ $\dfrac{1}{12}$　　④ $\dfrac{1}{9}$　　⑤ $\dfrac{5}{36}$

28. For a function $f(x)$ differentiable on the set of all real numbers, its derivative $f'(x)$ is given by

$$f'(x) = -x + e^{1-x^2}.$$

For a positive constant t, let $g(t)$ be the area of the region bounded by the tangent line to the curve $y = f(x)$ at the point $(t, f(t))$, the curve $y = f(x)$, and the y-axis. What is the value of $g(1) + g'(1)$? [4 points]

① $\dfrac{1}{2}e + \dfrac{1}{2}$　　② $\dfrac{1}{2}e + \dfrac{2}{3}$　　③ $\dfrac{1}{2}e + \dfrac{5}{6}$

④ $\dfrac{2}{3}e + \dfrac{1}{2}$　　⑤ $\dfrac{2}{3}e + \dfrac{2}{3}$

4   Mathematics Section (Calculus)

Short-Answer Questions

29. Let $\{a_n\}$ be a geometric sequence satisfying

$$\sum_{n=1}^{\infty}(|a_n|+a_n)=\frac{40}{3}, \quad \sum_{n=1}^{\infty}(|a_n|-a_n)=\frac{20}{3}$$

What is the sum of all natural numbers m satisfying the inequality $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{2n}\left((-1)^{\frac{k(k+1)}{2}}\times a_{m+k}\right)>\frac{1}{700}$? [4 points]

30. For two constants a $(1\le a\le 2)$ and b, the function $f(x)=\sin(ax+b+\sin x)$ satisfies the following conditions:

> (a) $f(0)=0$, $f(2\pi)=2\pi a+b$
>
> (b) The minimum positive value of t satisfying $f'(0)=f'(t)$ is 4π.

Let A be the set of all values of α in the open interval $(0, 4\pi)$ such that $f(x)$ has a local maximum at $x=\alpha$. Let n be the number of elements in set A, and let α_1 be the smallest element of A. If $n\alpha_1-ab=\frac{q}{p}\pi$, what is the value of $p+q$? (Note: p and q are coprime natural numbers.) [4 points]