10.

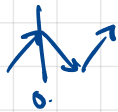
$$a_n = \begin{cases} 10. & (n \neq 3k). \\ -19. & (n = 3k). \end{cases} \qquad \sum_{k=1}^{n} a_k = \sum_{k=1}^{3n} a_k \ldots$$

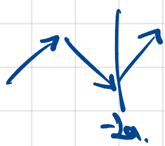
$S_{3n} = n.$ $\therefore S_n = S_{3n} = n.$

$S_{24} = 8.$

$S_n \ (n \geq 24)$ : $\overset{24}{8}.\ \overset{25}{18}.\ \overset{26}{28}.\ \overset{27}{9}.\ \overset{28}{19}.\ \overset{29}{29}.\ 10. \ldots$ $\therefore n = 29.$

11. $a \neq 0.$ $f(x) = x^3 + 3ax^2 + 4a.$ 극소 $-40\ldots$

$x = 0.$ 극대 $4a.$ $= -40.$ $a = -10.$  x.

$x = -2a.$ 극대 $4a^3 + 4a. = -40.$ $a = -2.$  ok.

$\therefore f(x) = x^3 - 6x^2 - 8.$ $f(2) = -24.$

12. $\cancel{\int_{-2}^{0} f(x)dx = \frac{1}{4}x^4+\frac{2}{3}x^3-\frac{1}{2}x^2+4x\Big|_{-2}^{0}}$

$\cancel{= -\left(4-\frac{16}{3}-2-8\right) = 6+\frac{16}{3} = \frac{40}{3}}$

접선. $-af'(a)+f(a) = -a(\underline{3a^2+4a-1})+(a^3+2a^2-a+4)$

$= -2a^3-2a^2+4=0.\quad a=1.\qquad f'(1)=6.$

$\int_{-2}^{1} f(x)dx = \left(\frac{1}{4}x^4+\frac{2}{3}x^3-\frac{1}{2}x^2+4x\right)\Big|_{-2}^{1}$

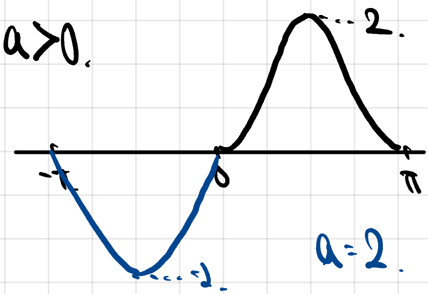
$= \left(\frac{1}{4}+\frac{2}{3}-\frac{1}{2}+\cancel{4}\right)-\left(\cancel{4}-\frac{16}{3}-2-8\right) = 16-\frac{1}{4}.$

$\therefore S = \left(16-\frac{1}{4}\right)-3 = \frac{51}{4}.$

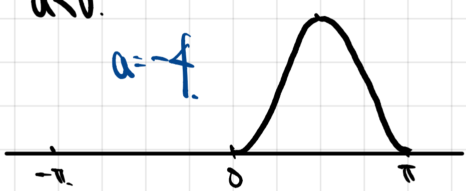
13. $a \neq 0.$ $\quad f(x) = \begin{cases} a\sin x & (x<0) \\ 1-\cos x & (x \geq 0) \end{cases}$

$[-\pi, \pi]$에서 최대 $M$. 최소 $m$. $M-m=4\ldots$

$a>0.$




$a<0.$




합. $-8$.

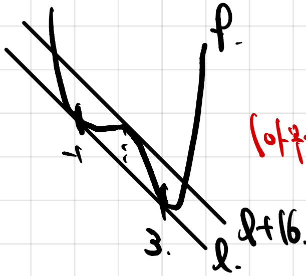
14. $f(x)=x^4+\cdots$

$x_1 \le x_2$. $\quad \int_{x_1}^{x_2} \{f(t)-f(a)\}dt \ge \int_{x_1}^{x_2} f'(a)(t-a)dt$.

$a \le -1$. or $a \ge 3$.

$f(1)=15$. $\quad f'(1)=1\ldots$

$\int_{x_1}^{x_2} \{f(t)-(x=a$에서 접선$)\}dt \ge 0$.



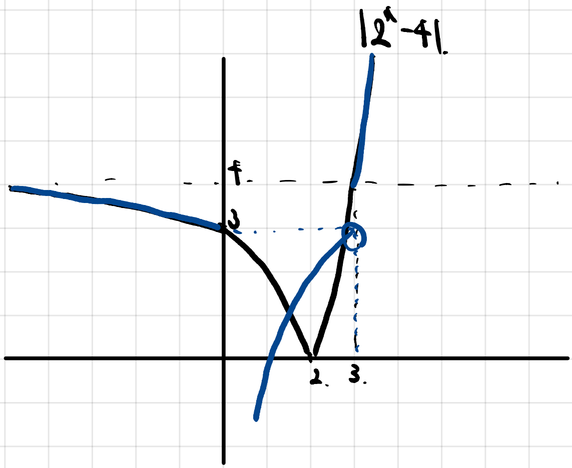

(아무거나 그려...)

$f(x)=(x+1)^2(x-3)^2+l(x)$

$=(x+1)^2(x-3)^2+(x-2)$ $\quad \therefore f(4)=27$.

15. $a$. $p$. $q$. $p<q$. $\quad f(x)=\begin{cases} |2^x-4| & (x\le p \text{ or } x\ge q) \\ a+\log_2 x & (p<x<q) \end{cases}$

$f: \mathbb{R} \to \mathbb{R}$ 1-1. onto...




(치역) $=\mathbb{R}$ $\quad p\le 0$.

(정의역) $=\mathbb{R}$ $\quad p\ge 0$. $\quad p=0$.

일대일 $\quad q\ge 3$.

$q>3$. $\{y \mid 4<y<2^q-4\}\ldots$

$q=3$.

$a+\log_2 3=3$. $\quad f\left(\frac{3}{2}\right)=3-1=2$.

19. $\int_0^x \{f(t)+t^2\}dt = xf(x) - x^3$. $\int_0^4 f'(t)dt$ ...

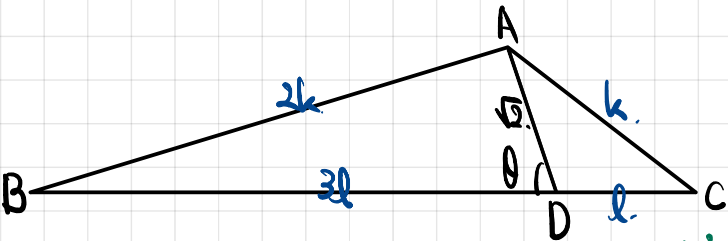
f 지분된 줄.

$x=0$. ...

미분. $f(x)+x^2 = f(x)+xf'(x)-3x^2$. $f'(x)=4x$. ...

$\therefore \int_0^4 = 32$.

*20. D. BC를 3:1로 내분. $AD=\sqrt{2}$. $AB:AC=2:1$. $\cos\theta = \frac{\sqrt{2}}{4}$ ...




$S = \left(\frac{AB}{\sin\theta} \times \frac{1}{2}\right)^2 \pi = \frac{8}{7} \times AC^2 \pi$ ... ~~X~~ $k^2$ 구하기.

$\frac{2+9l^2-4k^2}{2\cdot\cancel{\sqrt{2}}\cdot 3l} = \frac{\cancel{\sqrt{2}}}{4}$  $9l^2-4k^2+2=3l$

$\frac{2+l^2-k^2}{2\cdot\cancel{\sqrt{2}}\cdot l} = -\frac{\cancel{\sqrt{2}}}{4}$. $4l^2-4k^2+8=-4l$

$5l^2-6=7l$. $l=2$.

$\frac{8}{7}k^2 = \frac{8}{7}(l^2+l+2) = \frac{64}{7}$.  $7+64=71$

$l$ 안 구하고 $k^2$만 구하는 건... 되나?

21. $a_1 \in \mathbb{N}$. $a_{n+1} = \begin{cases} \frac{a_n}{n} & (a_n \geq 3) \\ 10 & (a_n < 3) \end{cases}$ $a_6 = 2$...

~~$a_4 < 3$. $a_5 = 10$. $a_6 = 2$ ✗.. $a_3 = 10$. $a_4 = \frac{10}{3}$ ✗. / $a_3 < 9$. $a_4 < 3$.~~

~~$a_4 \geq 3$. $a_5 = \frac{a_4}{4} = 10$. $a_4 = 40$.~~ ...

$a_5 = 10$... $a_4 = 40$. or $a_4 < 3$.

i) $a_4 = 40$ ; $a_3 = 120$. $a_2 = 240$. $a_1 = 240$.

ii) $a_4 < 3$ ; $\frac{a_3}{3} < 3$. $a_3 < 9$. $(a_3 \geq 3)$. $\therefore 3 \leq a_3 < 9$.

or $a_3 = 10$, $a_4 = \frac{10}{3}$. ✗.

$a_3 = \frac{a_2}{2}$. $6 \leq a_2 < 18$.

$a_2 = a_1$ $(a_1 \geq 3)$. $a_1 = 6, 7, \cdots, 17$

$a_2 = 10$ $(a_1 < 3)$. $a_1 = 1, 2$.

$\therefore \Sigma a_1 = 3 + 6 \cdot 23 + 240 = 381$...

수열의귀납적정의+가지치기 잘푸는법중.

22. $g(x) = \begin{cases} -f(x) & (x<0) \\ |f(x)| - |2x^2-8| & (x \ge 0) \end{cases}$ 미분가능...

$x=0$. $-f(0) = |f(0)| - 8$. $f(0) = 4$. $>0$.

$-f'(0) = f'(0)$ $f'(0) = 0$. $f(x) = ax^3 + bx^2 + 4$.

$x=2$. $|2x^2-8|$ 미분 x. $f(2) = 0$.

$0<x<2$에서 미분가능. $f(0)>0$. $|f(x)| = f(x)$

$x<2$. $f(x) - (8-2x^2)$.

$x>2$. $-f(x) - (2x^2-8)$

$f'(2) + 8 = 0$.

$f(2) = 8a + 4b + 4 = 0$ $2a+b+1=0$.

$f'(2) + 8 = 12a + 4b + 8 = 0$. $3a+b+2=0$.

$\therefore a=-1$ $b=1$. $f(x) = -x^3 + x^2 + 4$.

$f(-5) = 125 + 25 + 4 = 154$.

이지만

27. $a_n>0$.

$0<x<3$. $\sin\left(\frac{\pi}{a_n}x\right)=1$ 실근 2n개...

$0<x<\frac{3\pi}{a_n}$ $\sin x=1$ 실근 2n개.

$x=\frac{\pi}{2},\ \frac{5}{2}\pi,\ \frac{9}{2}\pi,\ \frac{13}{2}\pi,\ \cdots$

$(2+\frac{1}{2})\pi$, $(6+\frac{1}{2})\pi$, $(4n-\frac{3}{2})\pi$.

$\therefore (4n-\frac{3}{2})\pi<\frac{3\pi}{a_n}<(4n+\frac{1}{2})\pi$ $na_n\to\frac{3}{4}$ as $n\to\infty$.

28. $f(x)=ax^3+bx$ $(a>0)$.

$\forall x\in\mathbb{R}$ $\exists \lim_{n\to\infty}\frac{2x^{2n+2}+x^n+f(x)}{x^{2n}+x^n+1}$

$|x|<1$. $g(x)=f(x)$ $f(x)=\cancel{\frac{3\sqrt{3}}{2}}\,3\sqrt{3}(x^3-x)$.

$x=1$ $g(1)=1+\frac{1}{3}f(1)$ $g(-\frac{1}{2})\times g(2)=\left(\frac{3\sqrt{3}}{\cancel{2}}\cdot\frac{3}{8}\right)\times\cancel{16}\,8$

$=9\sqrt{3}$.

$|x|>1$. $g(x)=2x^2$.

$x=-1$ $\exists g(-1)$. $\begin{cases} n \text{ 짝수} & 1+\frac{1}{3}f(-1) \\ n \text{ 홀수} & 1+f(-1) \end{cases}$

$\therefore f(-1)=f(1)=0$. $b=-a$.

$k\geq3$. or $k=1$.

$\cancel{k>2}$ $g(x)=k$ 근 최대 2개. $\therefore \exists k=\cancel{1}\,2$ s.t. $g(x)=k$ 실근 1개.

30. $r \in \mathbb{Q}$. $a_n = a \cdot r^{n-1}$.

$\{a_n\}$의 항 중 한 자리 자연수 3개...

$9.\ 3.\ 1.$ $r = \frac{1}{3}$. $\cancel{3}$

$9.\ 6.\ 4.$ $r = \frac{2}{3}$. $\cancel{\frac{3}{2}}$. 더 없음.... 기분나쁨... 어떻게 앎 그걸...

$4.\ 2.\ 1.$ $r = \frac{1}{2}$ X. $\{a_n\}$ 수렴.

$$\lim_{n\to\infty} \frac{a_1 a_{n+1} + a_{2n}}{a_{n+1} + a_n} = \frac{ar}{r+1} = \frac{81}{10}$$

$$= \frac{a}{1 + \frac{1}{r}}$$

$r = \frac{1}{3}$. $\frac{a}{4} = \frac{81}{10}$ $a = \frac{162}{5}$. X.

$r = \frac{2}{3}$. $\frac{2}{5}a = \frac{81}{10}$. $a = \frac{81}{4}$. $\{a_n\}: \frac{81}{4}.\ \frac{27}{2}.\ 9.\ 6.\ 4.\ \cdots$. ok.

$r = \frac{1}{2}$. $\frac{a}{3} = \frac{81}{10}$. $a = \frac{27}{10}$. X.

$\therefore a_7 = 4 \times \left(\frac{2}{3}\right)^2 = \frac{16}{9}$ $9 + 16 = 25$.