06 2013학년도 9월 평가원(가형) 21번

최고차항의 계수가 1인 삼차함수 $f(x)$의 역함수를 $g(x)$라 할 때, $g(x)$가 다음 조건을 만족시킨다.

(가) $g(x)$는 실수 전체의 집합에서 미분가능하고 $g'(x) \le \frac{1}{3}$이다. $\longrightarrow f'(x) \ge 3$

(나) $\lim_{x \to 3} \frac{\overset{\to 0}{f(x) - g(x)}}{\underset{\to 0}{(x-3)g(x)}} = \frac{8}{9}$ $\longrightarrow f(3) - g(3) = 0 \quad \therefore f(3) = g(3)$

$f(1)$의 값은? [4점]

① -11 ② -9 ③ -7
④ -5 ⑤ -3

$$\lim_{x\to3} \frac{f(x)-g(x)}{(x-3)g(x)} = \lim_{x\to3} \frac{f(x)-f(3)-g(x)+g(3)}{(x-3)g(x)} \quad =3$$

$$= \lim_{x\to3} \left\{ \frac{f(x)-f(3)}{x-3} - \frac{g(x)-g(3)}{x-3} \right\} \times \frac{1}{g(x)}$$

$$= \frac{1}{3}\left\{ f'(3) - g'(3) \right\} = \frac{8}{9}$$

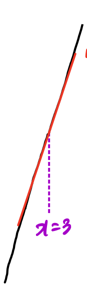
$$f'(3) - g'(3) = \frac{8}{3}$$

$$\frac{1}{g'(3)} - \underbrace{g'(3)}_{X} = \frac{8}{3}$$

$$3X^2 + 8X - 3 = 0 \longrightarrow (3X-1)(X+3) = 0$$

$$\therefore g'(3) = \frac{1}{3} \quad \left(\because 0 < g'(3) \le \frac{1}{3}\right)$$

$$\therefore f'(3) = 3$$



$$f(x) = (x-3)^3 + \cancel{a(x-3)^2} + b(x-3) + c \quad \because f''(3) = 0$$

$$= (x-3)^3 + \underset{3}{b}(x-3) + c \quad \because f'(3) = 3$$

$$= (x-3)^3 + 3(x-3) + \underset{3}{c} \quad \because f(3) = 3$$

$$= (x-3)^3 + 3(x-3) + 3$$

$$\therefore f(1) = -8 - 6 + 3 = -11$$