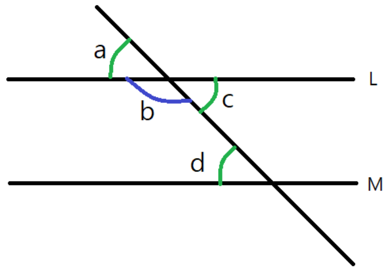
중학 수학 도형 정리 by 어차피 인생은 정시? [866752] 2020.12.31

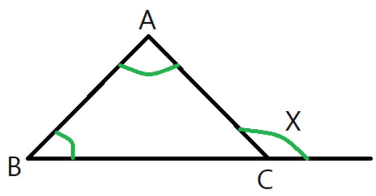
[1] 평행선의 성질



L // M

-> ∠ a = ∠ d , ∠ c = ∠ d , ∠ b + ∠ d = 180˚

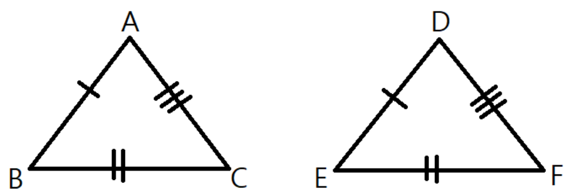
[2] 삼각형의 외각



∠ X = ∠ A + ∠ B

[3] 삼각형의 합동

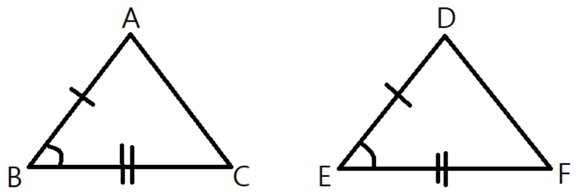
1) SSS 합동



Δ ABC ≡ Δ DEF (SSS 합동)

$\overline{AB} = \overline{DE}, \overline{BC} = \overline{EF}, \overline{CA} = \overline{FD}$

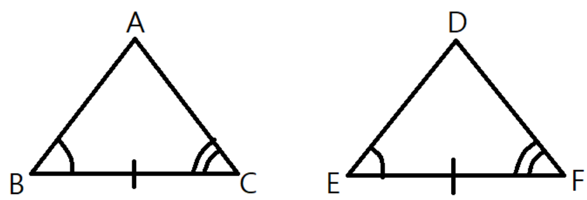
2) SAS 합동



Δ ABC ≡ Δ DEF (SAS 합동)

$\overline{AB} = \overline{DE}, \angle B = \angle E, \overline{BC} = \overline{EF}$

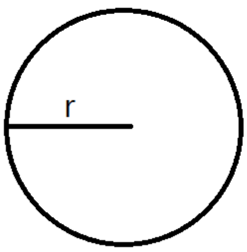
3) ASA 합동



Δ ABC ≡ Δ DEF (ASA 합동)

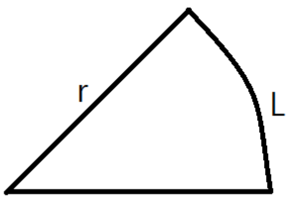
$\angle B = \angle E, \overline{BC} = \overline{EF}, \angle C = \angle F$

[4] 원, 부채꼴 길이와 넓이



원 둘레 = $2\pi r$

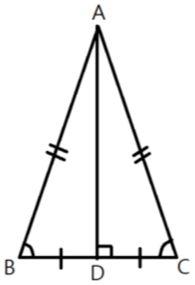
원 넓이 = πr^2



호의 길이 $L = r\theta$

호의 넓이 $S = \frac{1}{2}r^2\theta = \frac{1}{2}rL$

[5] 이등변삼각형의 성질

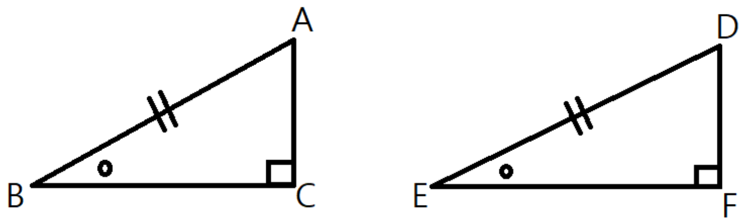


$\overline{AB} = \overline{AC}$, $\angle B = \angle C$

$\angle$A의 이등분선이 $\overline{BC}$를 수직이등분한다.

[6] 직각삼각형의 합동

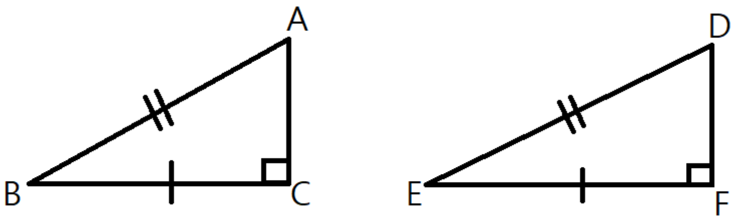
1) RHA 합동



Δ ABC ≡ Δ DEF (RHA 합동)

$\angle B = \angle E, \overline{AB} = \overline{DE}, \angle C = \angle F = 90°$

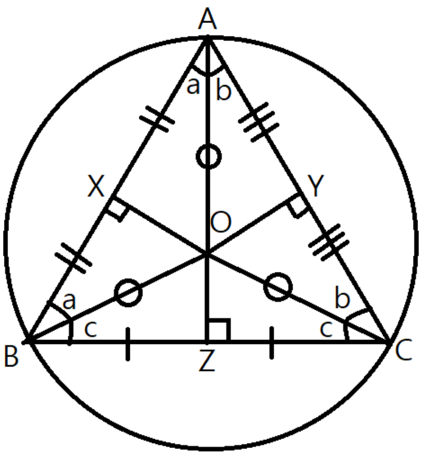
2) RHS 합동



Δ ABC ≡ Δ DEF (RHS 합동)

$\overline{AB} = \overline{DE}, \overline{BC} = \overline{EF}, \angle C = \angle F = 90°$

[7] 삼각형의 외심

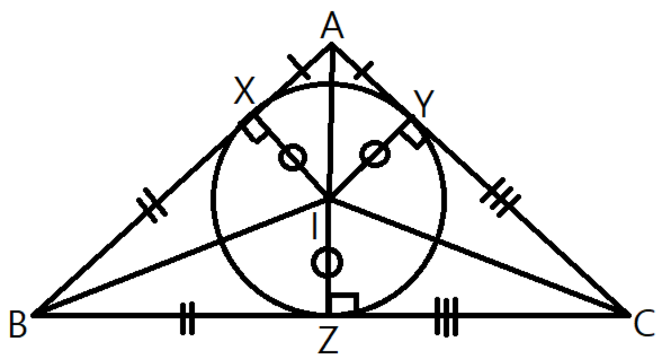


삼각형의 외접원의 중심 = O = 외심

$\overline{OA} = \overline{OB} = \overline{OC} = R$

외심: 세 변의 수직이등분선의 교점

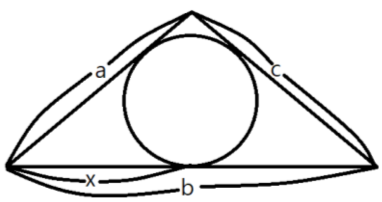
[8] 삼각형의 내심



삼각형의 내접원의 중심 = I = 내심

$\overline{IX} = \overline{IY} = \overline{IZ} = r$

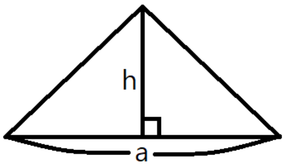
내심: 세 각의 이등분선의 교점



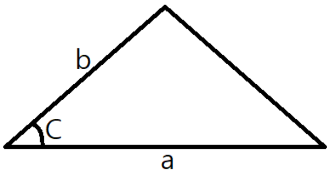
$x = \frac{a+b-c}{2}$

[9] 삼각형의 넓이

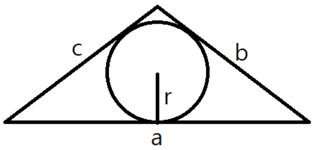
1) $S=\frac{1}{2}ah$



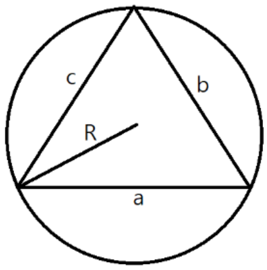
2) $S=\frac{1}{2}absinC$



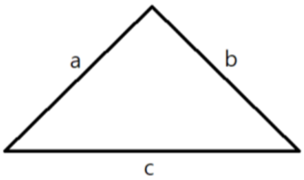
3) $S=\frac{1}{2}r(a+b+c)$



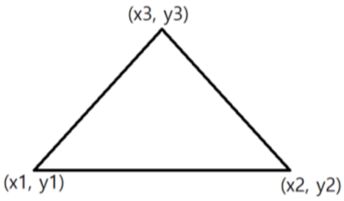
4) $S=\frac{abc}{4R}$



5) $S=\sqrt{t(t-a)(t-b)(t-c)}, t=\frac{a+b+c}{2}$

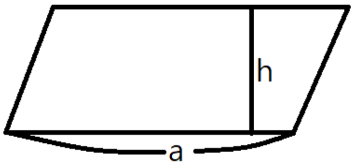


6) $S=\frac{1}{2}\begin{vmatrix} x1 & x2 & x3 \\ y1 & y2 & y3 \end{vmatrix}=\frac{1}{2}|x1y2+x2y3+x3y1-x2y1-x3y2-x1y3|$

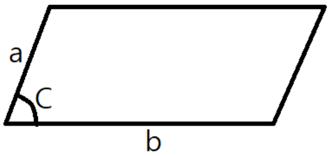


[10] 사각형의 넓이

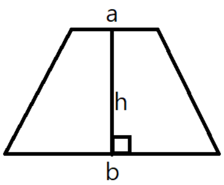
1) S=ah (평행사변형)



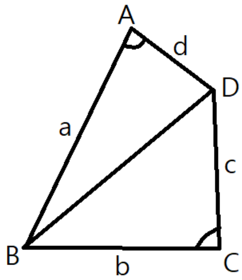
2) $S=absinC$ (평행사변형)



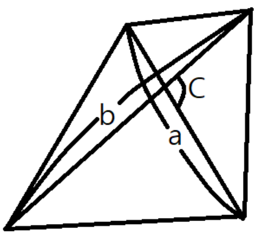
3) $S=\frac{1}{2}(a+b)h$



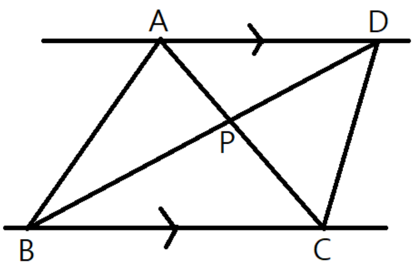
4) $S=\frac{1}{2}adsinA+\frac{1}{2}bcsinC$



5) $S=\frac{1}{2}absinC$ (임의의 사각형)



6) 등적변형

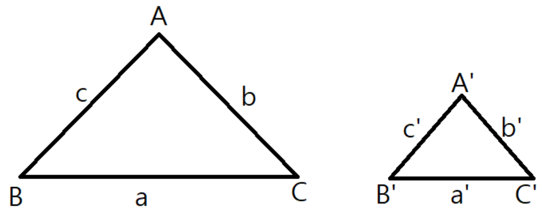


ΔABC=ΔDBC

ΔAPB=ΔDPC

[11] 삼각형의 닮음

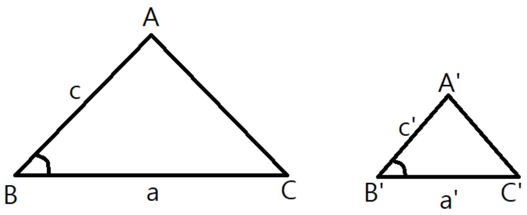
1) SSS 닮음



$\triangle$ ABC ∽ $\triangle$ A′B′C′ (SSS 닮음)

$a : a' = b : b' = c : c'$

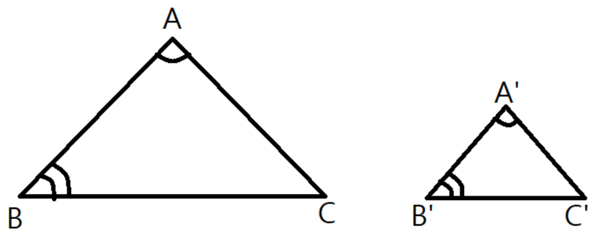
2) SAS 닮음



$\triangle$ ABC ∽ $\triangle$ A′B′C′ (SAS 닮음)

$a : a' = c : c'$, $\angle B = \angle B'$

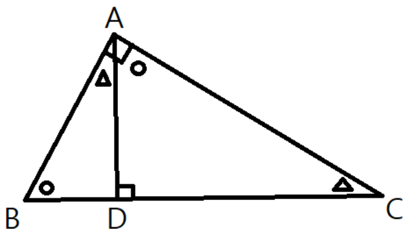
3) AA 닮음



$\triangle$ ABC ∽ $\triangle$ A′B′C′ (AA 닮음)

$\angle A = \angle A'$, $\angle B = \angle B'$

[12] 직각 삼각형의 닮음

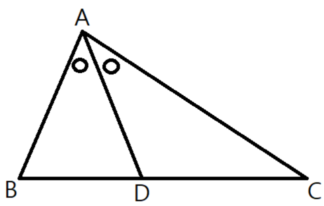


$\triangle$ ABC ∽ $\triangle$ DBA ∽ $\triangle$ DAC (AA 닮음)

$\overline{AB}^2 = \overline{BD} \cdot \overline{BC}, \overline{AC}^2 = \overline{CD} \cdot \overline{CB}, \overline{AD}^2 = \overline{BD} \cdot \overline{DC}$

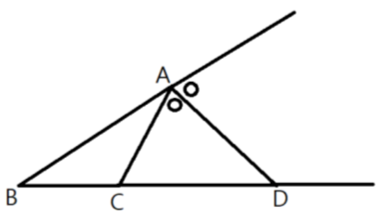
[13] 삼각형의 내각, 외각의 이등분선

1) 내각의 이등분선



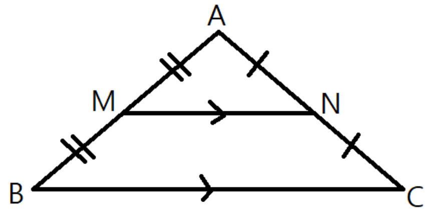
$\overline{AB} : \overline{AC} = \overline{BD} : \overline{CD}$

2) 외각의 이등분선



$\overline{AB} : \overline{AC} = \overline{BD} : \overline{CD}$

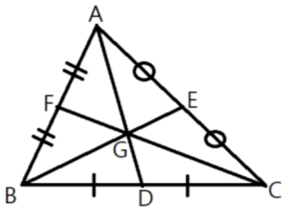
[14] 삼각형의 중점연결정리



$\overline{AM} = \overline{BM}\,, \overline{AN} = \overline{CN}$

-> $\overline{MN} // \overline{BC}$, $\overline{MN} = \frac{1}{2}\overline{BC}$

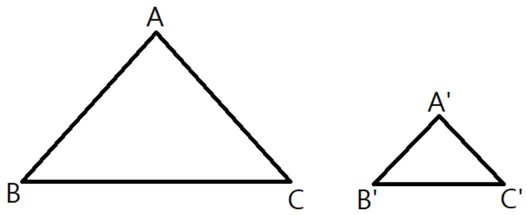
[15] 삼각형의 무게중심



무게중심: 세 중선의 교점 (중선: 한 꼭짓점과 그 대변의 중점을 연결한 선분)

$\overline{AG} : \overline{GD} = \overline{BG} : \overline{GE} = \overline{CG} : \overline{GF} = 2 : 1$

[16] 닮은 두 평면도형의 비



닮음비 a : b

-> 길이 비 = a : b

-> 넓이 비 = $a^2 : b^2$

[17] 삼각형의 유형 판별

1) 삼각형의 성립 조건: 세 변의 길이 a, b, c에 대해 $a \leq b \leq c$ -> $a + b > c$

2) 삼각형의 유형 판별과 피타고라스 정리 (세 변의 길이 a, b, c에 대해 $a \leq b \leq c$)

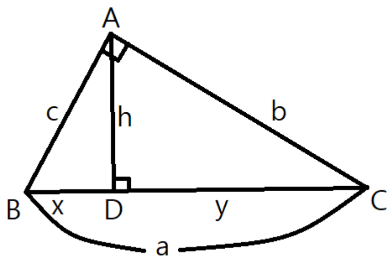
(1) $c^2 < a^2 + b^2$ -> $\angle C < 90^\circ$, Δ ABC : 예각삼각형

(2) $c^2 = a^2 + b^2$ -> $\angle C = 90^\circ$, Δ ABC : 직각삼각형

(3) $c^2 > a^2 + b^2$ -> $\angle C > 90^\circ$, Δ ABC : 둔각삼각형

[18] 피타고라스 정리의 활용

1) 직각 삼각형에서의 활용

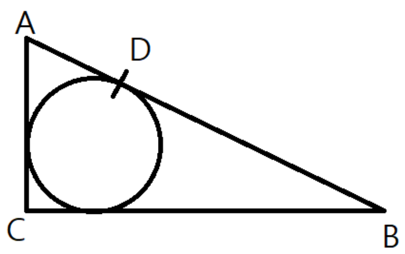


(1) 직각삼각형 넓이 -> bc = ah

(2) $c^2 : b^2 = x : y$

(3) $c^2 - b^2 = x^2 - y^2$

2) 직각 삼각형의 넓이

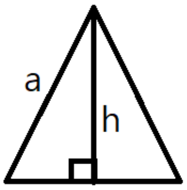


Δ ABC = $\overline{AD} \times \overline{BD}$

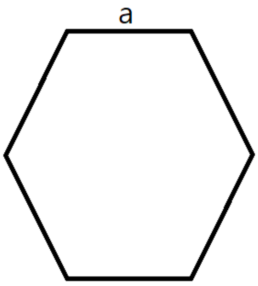
[19] 삼각비와 넓이

1) 정삼각형의 높이 $h = \frac{\sqrt{3}}{2}a$

넓이 $S = \frac{\sqrt{3}}{4}a^2$

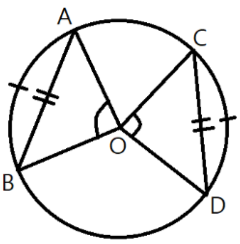


2) 정육각형의 넓이 $S = \frac{3\sqrt{3}}{2}a^2$



[20] 원과 직선

1)



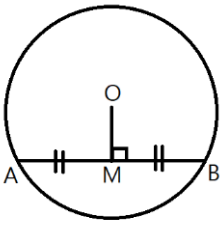
$\angle$ AOB = $\angle$ COD

$\Leftrightarrow$ $\widehat{AB} = \widehat{CD}$

$\Leftrightarrow$ $\overline{AB} = \overline{CD}$

* 중심각의 크기, 호의 길이는 정비례 O, 중심각의 크기, 현의 길이는 정비례 X

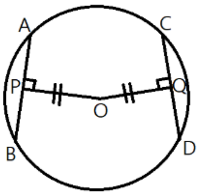
2)



$\overline{OM} \perp \overline{AB}\, , \overline{MA} = \overline{MB}$

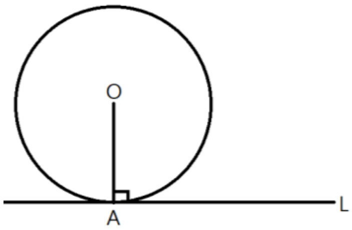
원의 중심에서 현에 내린 수선은 그 현을 수직이등분한다.

3)



$\overline{OP} = \overline{OQ} \Leftrightarrow \overline{AB} = \overline{CD}$

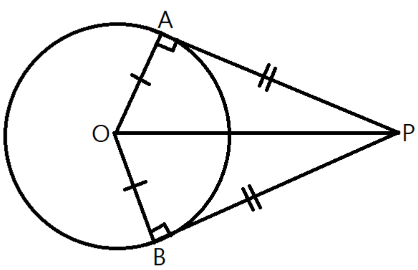
4)



$\overline{OA} \perp L$

원의 접선은 접점과 원의 중심을 연결한 선분에 수직이다.

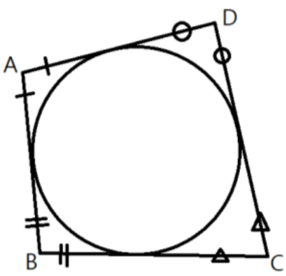
5)



$\overline{PA} = \overline{PB}$

원 외부의 한 점에서 그 원에 그은 두 접선의 길이는 서로 같다.

6)

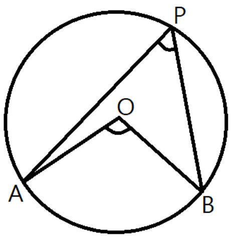


$\overline{AB} + \overline{CD} = \overline{AD} + \overline{BC}$ $\Leftrightarrow$ 사각형이 원에 외접

원에 외접하는 사각형에서 두 쌍의 대변의 길이의 합은 서로 같다.

[21] 원주각

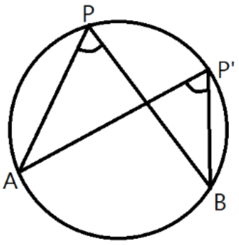
1)



$\angle AOB = 2\angle APB$

(중심각) = 2 (원주각)

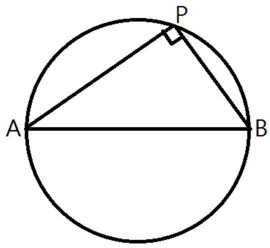
2)



$\angle APB = \angle AP'B$

한 원에서 한 호에 대한 원주각의 크기는 서로 같다.

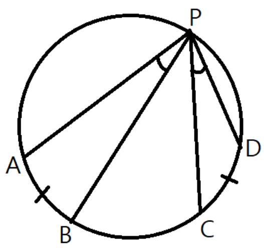
3)



$\overline{AB}$가 원의 지름 -> $\angle APB = 90^\circ$

지름의 원주각 = 90°

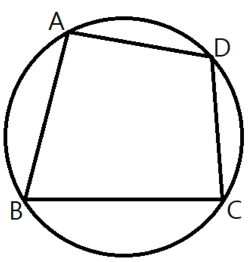
4)



호 길이 같음 ⇔ 원주각 크기 같음

$\overset{\frown}{AB} = \overset{\frown}{CD}$ ⇔ $\angle APB = \angle CQD$

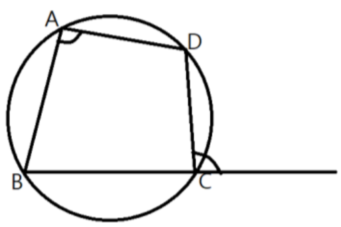
5)



대각의 크기의 합 = 180°

$\angle A + \angle C = 180^\circ$, $\angle B + \angle D = 180^\circ$

6)

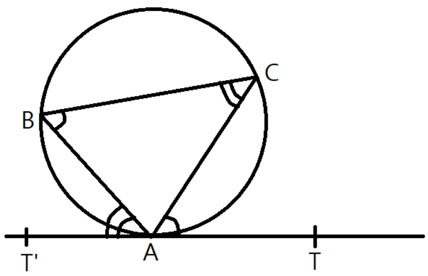


외각 크기 = 이웃한 내각과 마주보는 각의 크기

$\angle DCE = \angle A$

“외각 = 대내각”

7)



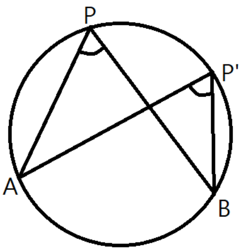
$\angle CAT = \angle CBA$ “접현각 = 원주각”

$\angle BAT' = \angle BCA$

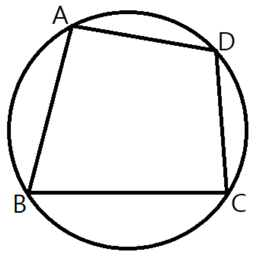
접선과 현이 이루는 각의 크기 = 현의 원주각의 크기

[22] 내접사각형: 네 점이 한 원 위에 있을 조건

1) 원주각 동일 -> (A, P, P′, B) 한 원 위

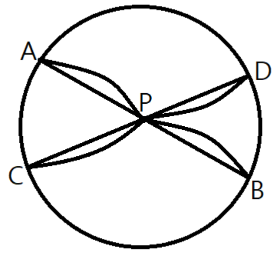


2) 대각의 크기의 합 180˚ -> (A, B, C, D) 한 원 위



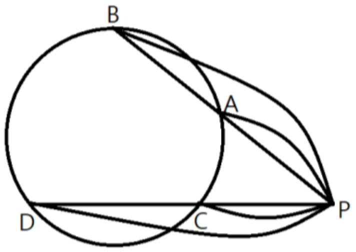
[23] 원과 비례

1)



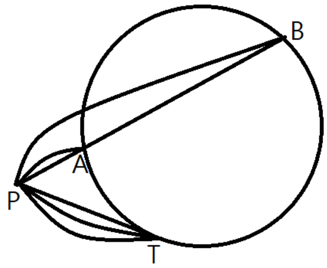
$\overline{PA}\cdot\overline{PB}=\overline{PC}\cdot\overline{PD}$

2)



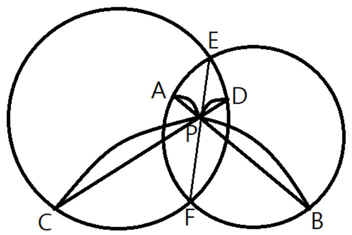
$\overline{PA}\cdot\overline{PB}=\overline{PC}\cdot\overline{PD}$

3)



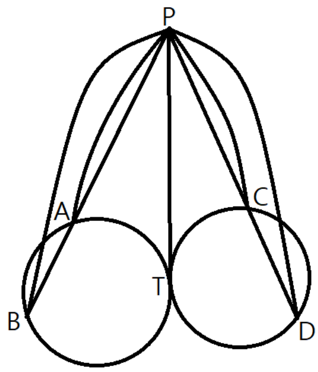
$\overline{PT}^2=\overline{PA}\cdot\overline{PB}$

4)



$\overline{PA}\cdot\overline{PB}=\overline{PC}\cdot\overline{PD}\ (=\overline{PE}\cdot\overline{PF})$

5)



$\overline{PA}\cdot\overline{PB}=\overline{PC}\cdot\overline{PD}\ (=\overline{PT}^2)$