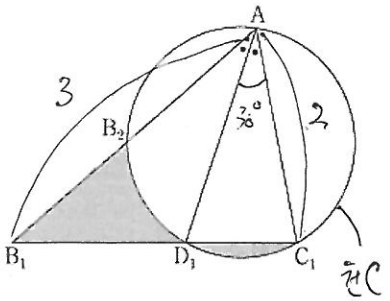
* 2021 학년도 평가원 6월 수학 가형 20번.



R_1

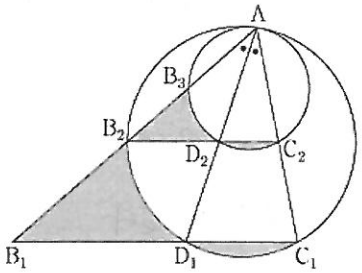
1) $n: 2 \to 2$. $\therefore n=1$.

2) l_r : $\overline{AB_1} \to \overline{AB_2}$.

$\angle A = \frac{\pi}{3} = 60°$. $\therefore \overline{B_1C_1}^2 = 9+4-2\cdot3\cdot2\cdot\frac{1}{2} = 7$. $\therefore \overline{B_1C_1} = \sqrt{7}$.

$\therefore$ 각의 이등분선의 성질에 의해 $\overline{B_1D_1} = \frac{3\sqrt{7}}{5}$. $\overline{D_1C_1} = \frac{2\sqrt{7}}{5}$.

R_1에 있는 $\angle AD_1C_1$의 외접원을 원C라 하고 반지름을 r_1이라 하면 $\frac{\overline{D_1C_1}}{\sin A = \sin(\angle D_1AC_1)} = 2r = \frac{2\sqrt{7}/5}{1/2}$. $\therefore r = \frac{2\sqrt{7}}{5}$.

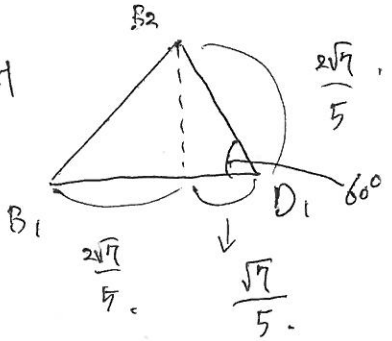


R_2

⋮ ⋮

$\angle B_2AD_1 = \angle D_1AC_1$ 이므로 $\overline{B_2D_1} = \overline{D_1C_1} = \frac{2\sqrt{7}}{5}$. $\angle B_2AC_1 = 60°$. $\therefore \angle B_2D_1C_1 = 120°$

$\triangle B_1D_1B_2$ 에서



$\therefore \overline{B_1B_2}^2 = \frac{28}{25} + \frac{63}{25} - 2\cdot\frac{2\sqrt{7}}{5}\cdot\frac{3\sqrt{7}}{5}\cdot\frac{1}{2}$

$= \frac{28+63-42}{25} = \frac{49}{25}$. $\therefore \overline{B_1B_2} = \frac{7}{5}$.

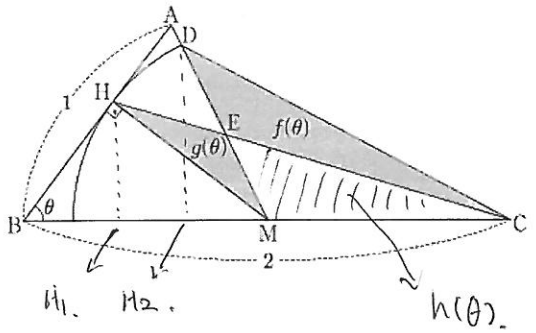
따라서 $\overline{AB_2} = \overline{AB_1} - \overline{B_1B_2} = 3 - \frac{7}{5} = \frac{8}{5}$ $\therefore l_r : \frac{8/5}{3} = \frac{8}{15}$. $S_r = \frac{64}{225}$.

3) $\overset{\frown}{D_1C_1}$과 $\overset{\frown}{B_2D_1}$의 2두 중심각 60°, 원주각 30°이므로 그 활꼴의 넓이 또한 같으므로 R_1에 색칠된 두 부분의 넓이를 합하면 $\triangle B_1D_1B_2$가 된다.

$\therefore a = \frac{1}{2}\times\overline{B_1D_1}\times\overline{D_1B_2}\times\sin 60° = \frac{1}{2}\times\frac{3\sqrt{7}}{5}\times\frac{2\sqrt{7}}{5}\times\frac{\sqrt{3}}{2} = \frac{21\sqrt{3}}{50}$.

따라서 $\lim_{n\to\infty} S_n = \frac{a}{1-r} = \frac{\frac{21\sqrt{3}}{50}}{1-\frac{64}{225}} = \frac{\frac{21\sqrt{3}}{50}}{\frac{161}{225}} = \frac{27\sqrt{3}}{46}$ //

* 2021 학년도 평가원 6월 수학 가형 28번.



$\overline{BM}=1$. $\therefore \triangle ABM$은 이등변삼각형.

$\therefore \angle BMA=\angle MAB=\frac{\pi-\theta}{2}=\frac{\pi}{2}-\frac{\theta}{2}$

$\angle BMH=\frac{\pi}{2}-\theta$. $\therefore \angle HMA=\frac{\theta}{2}$.

점 H의 직선 BC 위로의 수선의 발을 H_1, 점 D의 수선의 발을 H_2.

$\overline{BH}=\cos\theta$, $\overline{HH_1}=\cos\theta\sin\theta$, $\overline{DH_2}=\overline{DM}\times\sin(\angle MDH_2)=\sin\theta\times\sin\left(\frac{\pi}{2}-\frac{\theta}{2}\right)=\sin\theta\cdot\cos\frac{\theta}{2}$.

$\triangle MCE=h_1(\theta)$라 하면 $f(\theta)-g(\theta)=\{f(\theta)+h(\theta)\}-\{g(\theta)+h(\theta)\}$ 차 같다.

(추가적으로 선분 ME, $\angle MEH=\angle DEC$ 등을 구해서 $f(\theta)$, $g(\theta)$를 따로 구하는 것은 시험장 내에서는 비현실적이므로 $\triangle MCE$를 활용하는 방안을 생각해야 한다)

$\therefore \triangle HMC=f(\theta)+h(\theta)=\frac{1}{2}\times1\times\cos\theta\times\sin\theta$

$\triangle DMC=f(\theta)+h(\theta)=\frac{1}{2}\times1\times\cos\frac{\theta}{2}\times\sin\theta$

$\therefore f(\theta)-g(\theta)=\frac{1}{2}\times\left\{\sin\theta\cdot\cos\frac{\theta}{2}-\sin\theta\cdot\cos\theta\right\}$

$$\therefore \lim_{\theta\to0+}\frac{f(\theta)-g(\theta)}{\theta^3}=a=\lim_{\theta\to0+}\frac{\frac{1}{2}\sin\theta\times\left\{\cos\frac{\theta}{2}-\cos\theta\right\}}{\theta^3}=\lim_{\theta\to0+}\frac{1}{2}\times\frac{\sin\theta}{\theta}\times\frac{1}{\theta^2}\times\left(\cos\frac{\theta}{2}-\cos\theta\right)$$

$$=\lim_{\theta\to0+}\frac{1}{2}\times\frac{\sin\theta}{\theta}\times\frac{1}{\theta^2}\times\left(1-\cos\theta-1+\cos\frac{\theta}{2}\right)=\lim_{\theta\to0+}\frac{1}{2}\cdot\frac{\sin\theta}{\theta}\cdot\left(\frac{1-\cos\theta}{\theta^2}-\frac{1-\cos\frac{\theta}{2}}{\theta^2}\right)$$

$$=\lim_{\theta\to0+}\frac{1}{2}\times\frac{\sin\theta}{\theta}\times\left(\frac{\sin^2\theta}{\theta^2}\times\frac{1}{1+\cos\theta}-\frac{\sin^2\frac{\theta}{2}}{\frac{\theta^2}{4}}\times\frac{1}{4}\times\frac{1}{1+\cos\frac{\theta}{2}}\right)=\frac{1}{2}\times1\times\left(\frac{1}{2}-\frac{1}{8}\right)=\frac{3}{16}=a.$$

→ 반각공식 활용. $\cos\frac{\theta}{2}-\cos\theta=\cos\frac{\theta}{2}-\left(2\cos^2\frac{\theta}{2}-1\right)=-2\cos^2\frac{\theta}{2}+\cos\frac{\theta}{2}+1$

$=\left(2\cos\frac{\theta}{2}+1\right)\left(1-\cos\frac{\theta}{2}\right)$ $\therefore \lim_{\theta\to0+}\frac{1}{2}\times\frac{\sin\theta}{\theta}\times\left(2\cos\frac{\theta}{2}+1\right)\times\frac{\sin^2\frac{\theta}{2}}{\frac{\theta^2}{4}}\times\frac{1}{4}\times\frac{1}{1+\cos\frac{\theta}{2}}=\frac{3}{16}=a$.

$\therefore 80a=80\times\frac{3}{16}=15$