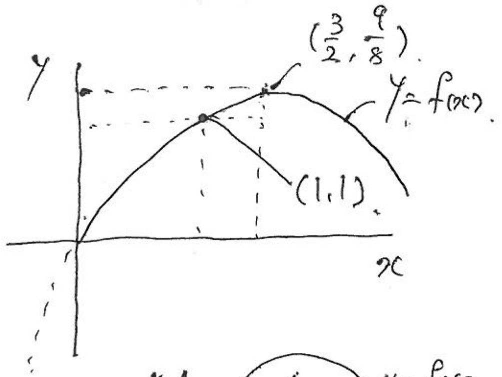
* 2018학년도 대수능 수학 나형 30번.

함수 $f(x) = \dfrac{3x-x^2}{2} = \dfrac{x(3-x)}{2}$, $D=[0,\infty)$



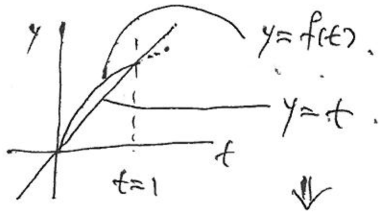
(가) $0 \le x < 1$, $g(x)=f(x)$.

(나) $n \le x < n+1$, $g(x)=\dfrac{1}{2^n}\{f(x-n)-(x-n)\}+x$

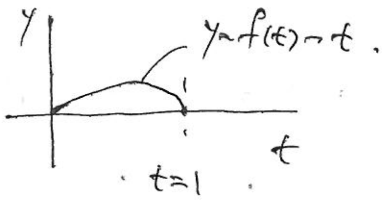
(단, n은 자연수).

(나)-1. $f(x-n)-(x-n) = f(t)-t$.

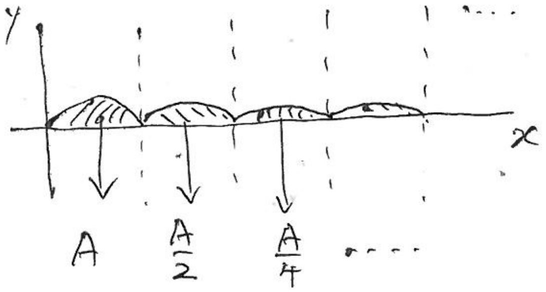
($x=n$일 때 $t=0$, $x \to n+1$일 때 $t \to 1$)



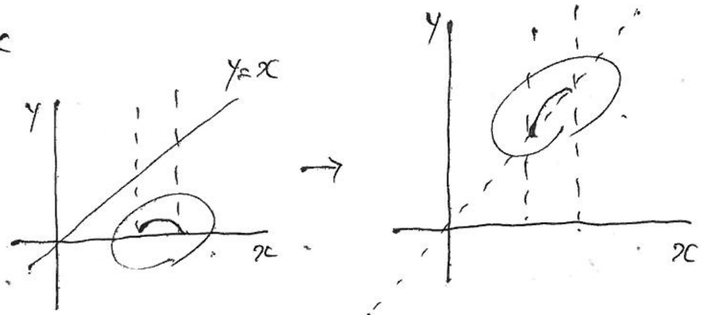
∴



(나)-2. $\dfrac{1}{2^n}\times\{f(x-n)-(x-n)\}$



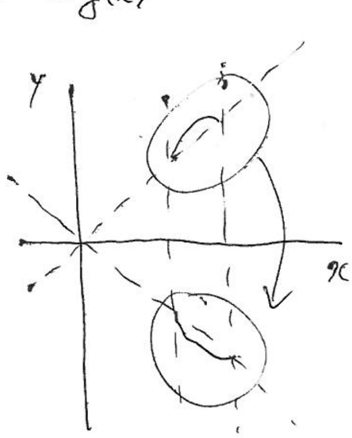
(나)-3. $\dfrac{1}{2^n}\times\{f(x-n)-(x-n)\}+x$.



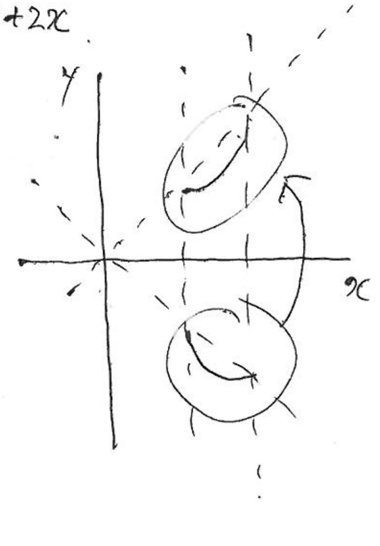
$y=x$ 그래프 위에 (나)-2를 더하는 그래프이다.

$\rightarrow h(x) = \begin{cases} g(x) & (0 \le x < 5 \text{ or } x \ge k) \\ 2x-g(x) & (5 \le x < k) \end{cases}$.

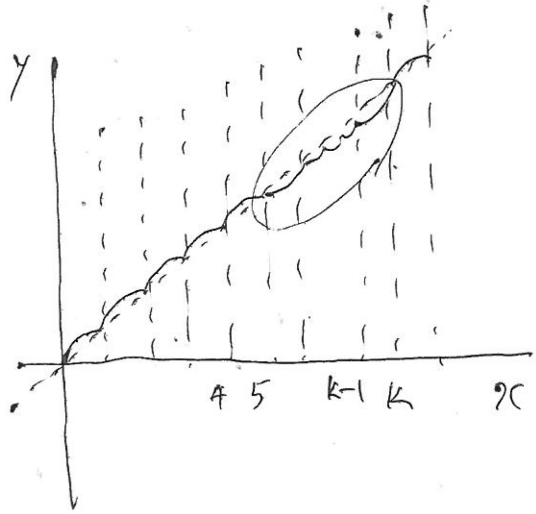
① $-g(x)$



② $+2x$



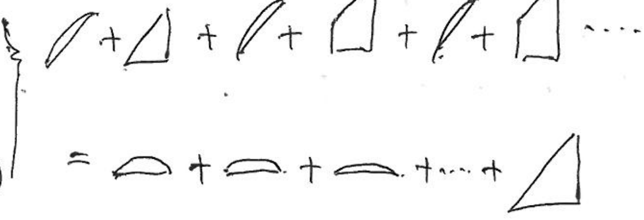
따라서 h(x)의 개형은 다음과 같다.



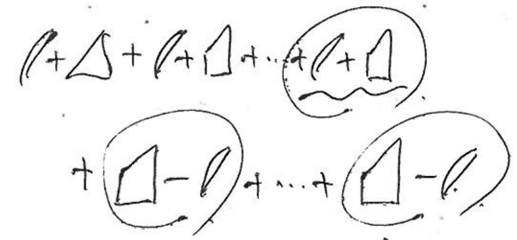
$a_n = \int_0^n h(x)\,dx$.

(n>k 인 경우)

$a_1 \sim a_5$ or $(a_{k+1} \sim a_n)$

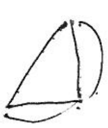


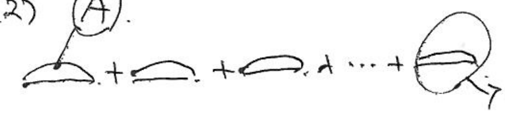
$a_6 \sim a_k$

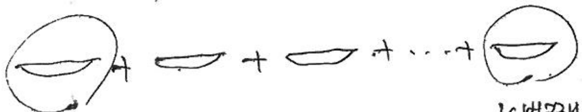


(n=6부터) (n=k까지)

∴ a_n은 3개 부분으로 나누어서 계산 가능.

(1)  n. n.

(2) A  → n번째.

(3)  ⇒ 개수는 k−5.

6번째. k번째.

(1)+(2)−(3)×2

$$A=\int_0^1 f(x)\,dx-\frac{1}{2}\times 1^2=\int_0^1\left\{\frac{3}{2}x-\frac{1}{2}x^2\right\}dx-\frac{1}{2}=\frac{3}{4}-\frac{1}{6}-\frac{1}{2}=\frac{1}{12}.$$

$$\lim_{n\to\infty}(2a_n-n^2)=\lim_{n\to\infty}2\left(a_n-\frac{n^2}{2}\right) \Rightarrow (2)\times 2-(3)\times 4$$

왜 이렇게 계산되는지 생각할 것.

$$\therefore\ 2\times\frac{A}{1-\frac{1}{2}}-4\times\frac{\frac{A}{32}\left(1-\left(\frac{1}{2}\right)^{k-5}\right)}{1-\frac{1}{2}}=\frac{241}{768}=4A-\frac{A}{4}+\frac{A}{4}\times\left(\frac{1}{2}\right)^{k-5}=\frac{15}{4}A+\frac{A}{4}\left(\frac{1}{2}\right)^k\cdot 32$$

$$\therefore\ \frac{15}{4}\times\frac{1}{12}+\frac{8}{12}\cdot\left(\frac{1}{2}\right)^k=\frac{241}{768}.\qquad \therefore\ \frac{2}{3}\cdot\left(\frac{1}{2}\right)^k=\frac{241}{768}-\frac{15}{48}=\frac{241}{768}-\frac{240}{768}=\frac{1}{768}$$

$$\left(\frac{1}{2}\right)^k=\frac{1}{768}\times\frac{3}{2}=\frac{1}{256}\times\frac{1}{2}=\frac{1}{512}.\qquad \therefore\ k=9 /\!/$$