* 2018년 4월 시행 교육청 모의고사 23수학 나형 30번.

실수 a, b. $D=\{x \mid x \geq 0\}$, $f(x)=\dfrac{-ax-b+1}{ax+b}$ $(ab>0) \rightarrow (a>0, b>0)$ or $(a<0, b<0)$

실수 k, $D=\{x \mid x \geq 0\}$, $g(x)=\begin{cases} 2k-f(x) & (f(x)<k) \\ f(x) & (f(x) \geq k) \end{cases}$ $y=k$를 기준으로 윗부분을 접어서 올린 형태의 그래프 ($y=k$ 대칭 X).

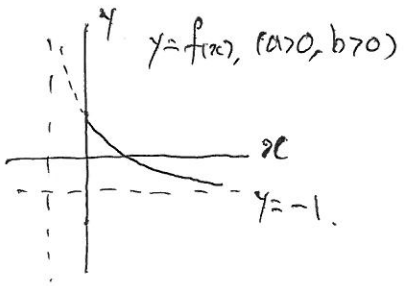
(가) $\lim_{x\to\infty} |g(x)|=\frac{1}{2}$.

(나) $|g(0)|=1$

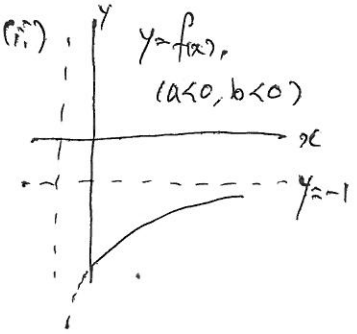
(다) $|g(x)|=-k$의 실근은 $x=\frac{1}{28}, \alpha$ $(\alpha>\frac{1}{28})$ → 서로 다른 실근 2개만.

$f(x)$의 점근선을 보면 $x=-\frac{b}{a}(<0)$, $y=-1$.

(i)



(ii)

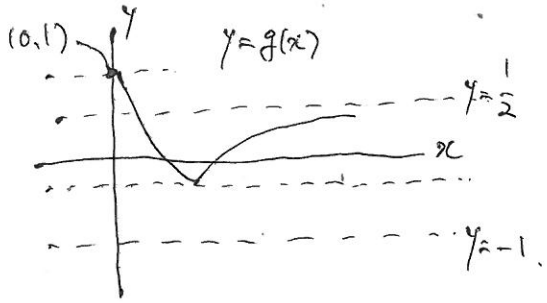


(가) $\lim_{x\to\infty} |g(x)|=\frac{1}{2}$

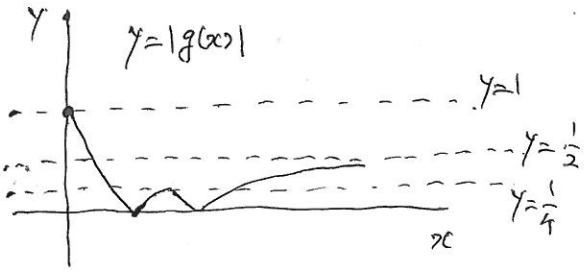
→ $\lim_{x\to\infty} g(x)=\frac{1}{2}$ or $-\frac{1}{2}$

(i) 개형에서 $k=-\frac{1}{4}$ $(\lim=\frac{1}{2})$, $k=-\frac{3}{4}$ $(\lim=-\frac{1}{2})$, (ii) 개형에서 $k=-\frac{3}{4}$ $(\lim=-\frac{1}{2})$, $k=-\frac{1}{4}$ $(\lim=\frac{1}{2})$

① $a>0, b>0, k=-\frac{1}{4}$

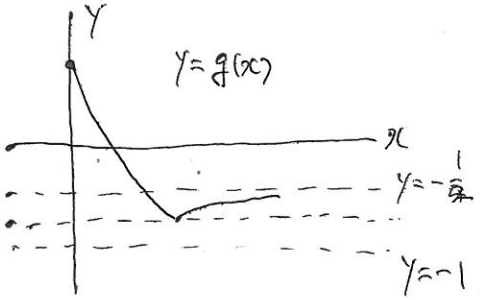


→

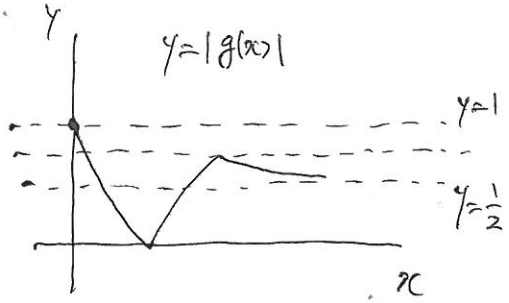


→ $y=|g(x)|$와 $y=-k=\frac{1}{4}$과 서로 다른 실근 3개.

② $a>0, b>0, k=-\frac{3}{4}$.



→

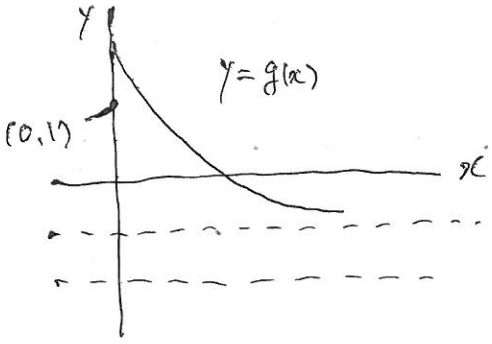


→ 서로 다른 실근 2개.

⇓

조건 만족.

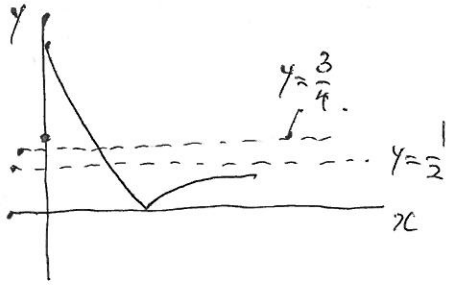
③ $a<0$, $b<0$, $k=-\frac{3}{4}$.



$|g(0)|>1$ 이므로

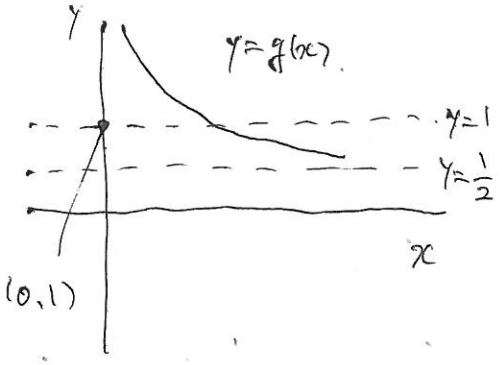
더 확인할 필요는

없지만 확인하자면

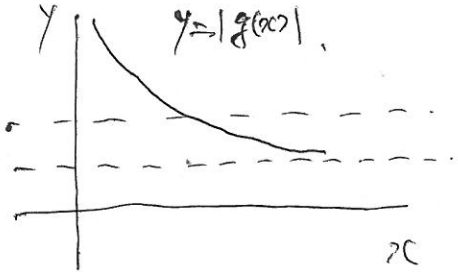


→ 서로 다른 실근 1개.

④ $a<0$, $b<0$, $k=-\frac{1}{4}$



→

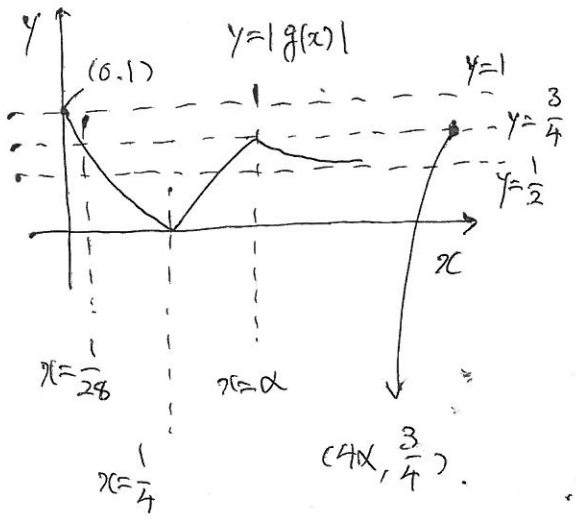


$g(x)=|g(x)|$.

$|g(0)|>1$.

서로 다른 실근 0개.

따라서 조건을 충족시키는 개형은 ② 이고 $f(x)=-1+\frac{1}{ax+b}$



$f(0)=1$ 에서 $\frac{1}{b}=2$ $\therefore b=\frac{1}{2}\ (>0)$

$f(\frac{1}{28})=\frac{3}{4}$ 이므로 $\frac{1}{\frac{a}{28}+\frac{1}{2}}=\frac{4}{3}$.

$\therefore \frac{a}{4}+\frac{4}{2}=4$ 에서 $a=2\ (a>0)$

$\therefore f(x)=-1+\frac{1}{2x+\frac{1}{2}}$ ($\because x$절편은 $\frac{1}{4}$).

$\therefore -1+\frac{1}{2\alpha+\frac{1}{2}}=-\frac{3}{4}$ 에서

$1=\frac{3}{4}=\frac{1}{4}=\frac{1}{2\alpha+\frac{1}{2}}$ $\therefore 2\alpha=\frac{7}{2}$.

$\therefore \alpha=\frac{7}{4}$

m의 값은 모두 3개.

㉠ $(0,1)\sim(7,\frac{3}{4})$. $m=-\frac{1}{28}$

㉡ $(\frac{1}{28},\frac{3}{4})\sim(7,\frac{3}{4})$ $m=0$

㉢ $(\frac{1}{4},0)\sim(7,\frac{3}{4})$. $m=\frac{1}{9}$.

$\therefore 252\times\left(-\frac{1}{28}+\frac{1}{9}\right)=-9+28=19$ //