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사차함수 $f(x)$ (실근의 개수는 최대 4), $n=1, 2, 3, 4, 5$

(가) $\sum_{k=1}^{n} f(k) = f(n) \times f(n+1)$ (나) $\frac{f(5)-f(3)}{5-3} \le 0$, $\frac{f(6)-f(4)}{6-4} \le 0$.

$l1: f(1) = f(1) \times f(2)$

$l2: f(1)+f(2) = f(2) \times f(3)$

$l3: f(1)+f(2)+f(3) = f(3) \times f(4)$

$l4: f(1)+f(2)+f(3)+f(4) = f(4) \times f(5)$

$l5: f(1)+f(2)+f(3)+f(4)+f(5) = f(5) \times f(6)$

$l1 \Rightarrow f(1) = 0$ / $f(1) \ne 0,\ f(2) = 1$.

↳ $l1-1$

$l2 \Rightarrow f(1)=f(2)=0$ / $f(1)=0,\ f(2)\ne 0,\ f(3)=1$.

↳ $l2-1$

$l3 \Rightarrow f(1)=f(2)=f(3)=0$ / $f(1)=f(2)=0,\ f(3)\ne 0,\ f(4)=1$.

↳ $l3-1$

$l4 \Rightarrow f(1)=\cdots=f(4)=0$ / $f(1)=f(2)=f(3)=0,\ f(4)\ne 0,\ f(5)=1$.

↳ * $l4-1$.

$l5 \Rightarrow f(1)=\cdots=f(4)=0,\ f(5)\ne 0,\ f(6)=1$.

$(f(6)>f(4) \rightarrow \times)$. $+\alpha$ $f(5)\ne 0$, 사차함수의 실근의 개수의 최댓값은 4이다.

* $l4-1$) $l5$를 거치면 $f(4)+1=f(6)$. $\rightarrow f(6)>f(4) \rightarrow (\times)$.

* $l3-1$) $l4$를 거치면 $f(3)+1=f(5) \rightarrow f(5)>f(3) \rightarrow (\times)$.

* $l2-1$) $l3$을 거치면 $f(2)+1=f(4)$, $f(1)=0,\ f(2)\ne 0,\ f(3)=1$.

$l4$에서 $2f(4) = f(4)\times f(5)$ $(\because f(1)+f(2)+f(3)+f(4) = 2f(4))$

$f(4)=0$ / $f(4)\neq 0,\ f(5)=2$.

$(f(1)=0,\ f(3)=1,\ f(2)=-1,\ f(4)=0)$ $(f(1)=0,\ f(3)=1,\ f(2)\neq 0,\ f(4)\neq 0,\ f(5)=2)$

$f(5)>f(3) \rightarrow \times$.

$\downarrow$

$f(5)=0$

$f(5)\neq 0,\ f(6)=1 \Rightarrow f(6)>f(4) \rightarrow \times$

$\downarrow$

가능성 존재 △1.

※ $l_1-1)$ $f(1)\neq 0,\ f(2)=1$.

l_2를 거치면 $f(1)+1=f(3)$.

l_3을 거치면 $f(1)+f(2)+f(3)=2f(3)=f(3)\times f(4)$

$f(3)=0,\ f(1)=-1,\ f(2)=1$ / $f(3)\neq 0,\ f(4)=2,\ f(1)\neq 0,\ f(2)=1$

$\downarrow$

l_4를 거치면 $f(4)=f(4)\times f(5)$.

l_4를 거치면 $2f(3)+2=2f(5)$.

$f(4)=0$ / $f(4)\neq 0,\ f(5)=1$ $(f(5)>f(3)\rightarrow\times)$.

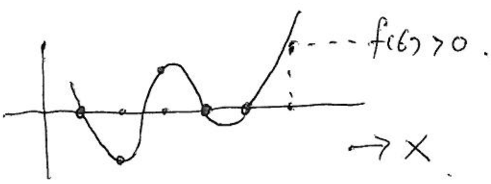
$f(5)>f(3) \rightarrow (\times)$.

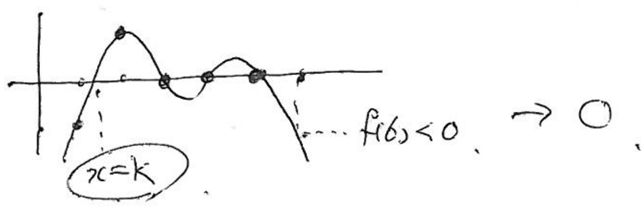
l_5를 거치면 $f(5)=f(5)\times f(6)$

$f(5)=0$ / $f(5)\neq 0,\ f(6)=1$. $(f(6)>f(4)\rightarrow\times)$

↳ $f(6)\leq 0$ 이면 가능성 존재 △2.

$\therefore$ △1 $f(1)=0,\ f(2)=-1,\ f(3)=1,\ f(4)=0,\ f(5)=0$. △2 $f(1)=-1,\ f(2)=1,\ f(3)=f(4)=f(5)=0,\ f(6)\leq 0$.





$\therefore f(x)=a(x-k)(x-3)(x-4)(x-5)$.

$f(1)=-24a(1-k)=-1=-24a+24ak$

$f(2)=-6a(2-k)=1.=-12a+6ak$

$\therefore\ 4=-48a+24ak$.

$24a=-5$. $a=-\frac{5}{24}$. $\therefore 5-5k=-1$ 에서 $k=\frac{6}{5}$.

따라서 $f(x)=-\frac{5}{24}(x-\frac{6}{5})(x-3)(x-4)(x-5)$

$f(\frac{5}{2})=\frac{65}{128}$. $\therefore 128\times f(\frac{5}{2})=65$//