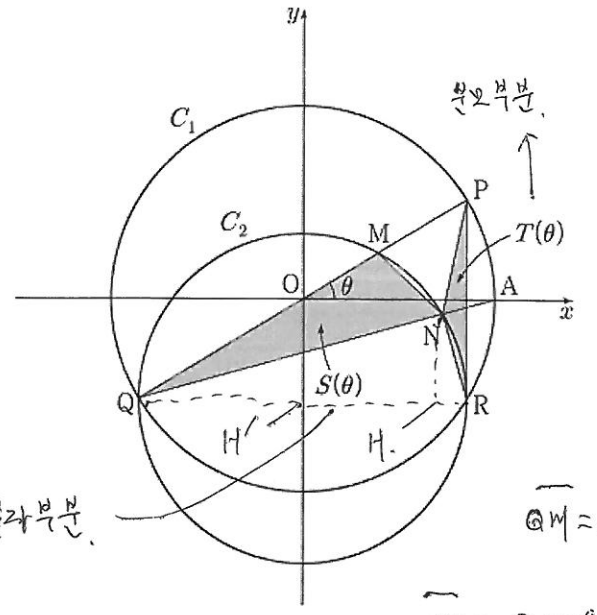
* 2018년 7월 시행 교육청 고3 수학 가형 21번.




O는 원점, A(1,0), P($\cos\theta$, $\sin\theta$)

Q($-\cos\theta$, $-\sin\theta$), R($\cos\theta$, $-\sin\theta$)

$\triangle OQA$는 이등변, $\therefore \angle OQA = \frac{\theta}{2} = \angle OAQ$.

($C_1$에서 호AP에 대한 중심각, 원주각으로도 확인가능)

$\overline{QR}$은 원 $C_2$의 지름이므로 $\angle QMR = \angle QNR = \frac{\pi}{2}$.

$\overline{QM} = 2\cos^2\theta$, $\overline{QN} = 2\cos\theta\cdot\cos\frac{\theta}{2}$.

$\overline{NR} = 2\cos\theta\cdot\sin\frac{\theta}{2}$. $\overline{QH} = 2\cos\theta\cdot\cos^2\frac{\theta}{2}$. $\overline{HR} = 2\cos\theta\left(1-\cos^2\frac{\theta}{2}\right)$ ($\sin^2\frac{\theta}{2}$).

$$\therefore \lim_{\theta\to 0+}\frac{\theta^2\times S(\theta)}{T(\theta)} = \lim_{\theta\to 0+}\frac{\theta^2\times\frac{1}{2}\times 2\cos^2\theta\times 2\cos\theta\cos\frac{\theta}{2}\cdot\sin\frac{\theta}{2}}{\frac{1}{2}\times 2\sin\theta\times 2\cos\theta\sin^2\frac{\theta}{2}} = \lim_{\theta\to 0+}\frac{\theta\times\frac{\theta}{2}\times 2\times\cos^2\theta\times\cos\frac{\theta}{2}}{\cos\theta\times\sin\theta\times\sin\frac{\theta}{2}} = 2 //$$

→ 좌표를 사용해서 신발끈 정리 (행렬식 차를 형태) 사용.

$\angle MH'R = 2\theta$. $\angle NH'R = \theta$. $\therefore$ M($\cos\theta\cos2\theta$, $\cos\theta\sin2\theta - \sin\theta$),

N($\cos\theta\cos\theta$, $\cos\theta\sin\theta - \sin\theta$).

$$T(\theta) = \frac{1}{2}\times 2\sin\theta\times(\cos\theta-\cos^2\theta) = \sin\theta\cdot\cos\theta\cdot(1-\cos\theta) = \frac{\cos\theta\cdot\sin^3\theta}{1+\cos\theta}$$ (분모부분).

$S(\theta)$ ⇒ (0,0) ($\cos\theta\cos2\theta+\cos\theta$, $\cos\theta\sin2\theta$), ($\cos^2\theta+\cos\theta$, $\cos\theta\sin\theta$) ← 행렬이용.

$$= \frac{1}{2}\times\left|\cos^2\theta\sin\theta(\cos2\theta+1) - \cos^2\theta\sin2\theta(\cos\theta+1)\right| \leftarrow (\sin2\theta = 2\sin\theta\cos\theta)$$

$$= \frac{1}{2}\cos^2\theta\sin\theta\left|\cos^2\theta-\sin^2\theta+1-2\cos^2\theta-2\right| = \frac{1}{2}\cos^2\theta\sin\theta\times\left|-\cos^2\theta-\sin^2\theta-1\right|$$

$$\therefore \lim_{\theta\to 0+}\frac{\theta^2\times S(\theta)}{T(\theta)} = \lim_{\theta\to 0+}\frac{\theta^2\times\frac{1}{2}\times\cos^2\theta\times\sin\theta\times 2}{\frac{\cos\theta\cdot\sin^3\theta}{1+\cos\theta}} = \frac{1}{\frac{1}{2}} = 2 //$$