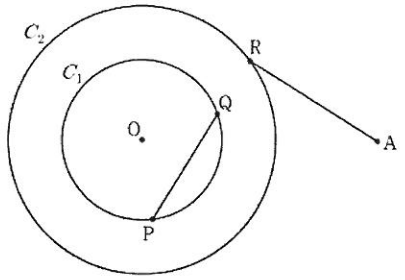
* 2018년7월 시행 교육청 고3 수학 가형 29번.



$\overline{OA}=2\sqrt{11}$, 원 C_1의 반지름 $\sqrt{5}$, 원 C_2의 반지름 $\sqrt{14}$.

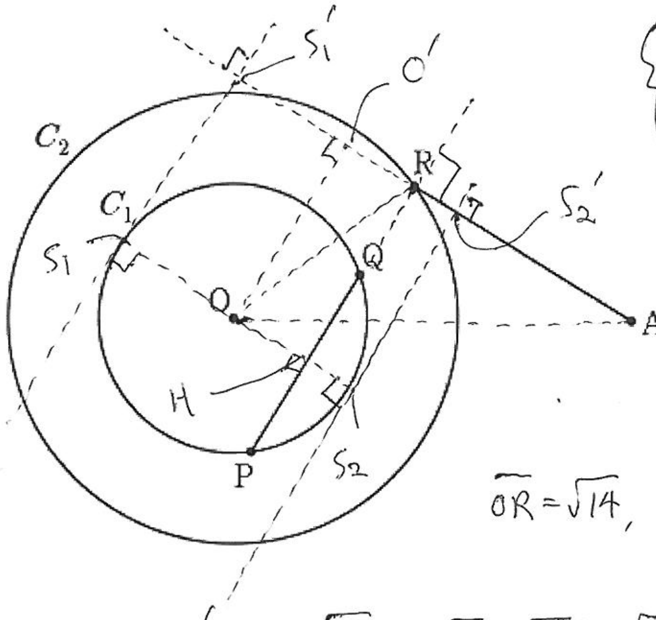
(가) $\overrightarrow{PQ}=k\cdot\overrightarrow{QR}$ $(k>0)$

(나) $\overrightarrow{PQ}\cdot\overrightarrow{AR}=0 \rightarrow \overrightarrow{PR}\cdot\overrightarrow{AR}=0$

$\left(\frac{\pi}{2}<\angle ORA<\pi\right)$

$\overline{PQ}:\overline{AR}=2:\sqrt{6} \rightarrow \overline{PQ}=2t,\ \overline{AR}=\sqrt{6}t$ $(t>0)$

→ $\overline{AR}$을 고정시키면 자동으로 $\overline{PQ}$도 고정된다.



$\overrightarrow{AR}\cdot\overrightarrow{AS}$의 최댓값 M은 $\overline{AR}\times\overline{AS_1'}$, 최솟값 m은 $\overline{AR}\times\overline{AS_2'}$

$\overline{PH}=t$라 할 때, $\overline{OP}=\sqrt{5}$, ∴ $\overline{OH}=\sqrt{5-t^2}$

∴ $\overline{O'R}=\sqrt{5-t^2}$,

$\overline{OR}=\sqrt{14}$, $\overline{OH}=\sqrt{5-t^2}$, ∴ $\overline{HR}=\sqrt{9+t^2}=\overline{OO'}$

∴ △OAO'에서 $\overline{OA}=2\sqrt{11}$, $\overline{OO'}=\sqrt{9+t^2}$, $\overline{AO'}=\sqrt{6}t+\sqrt{5-t^2}$.

∴ $44=9+t^2+6t^2+5-t^2+2\sqrt{30-6t^2}\cdot t$

$30-6t^2=2t\sqrt{30-6t^2} \rightarrow (30-6t^2)^2=4t^2(30-6t^2)$

∴ $30-6t^2=0$ or $4t^2$. → $t=\sqrt{5}$ or $\sqrt{3}$ 에서 $(0<t<\overline{OP}=\sqrt{5})$이므로 $t=\sqrt{3}$.

→ $\overline{OH}=\overline{O'R}=\sqrt{2}$, $\overline{AR}=\sqrt{18}=3\sqrt{2}$, $\overline{RS_2'}=\sqrt{5}-\sqrt{2}$, $\overline{OS_1}=\overline{O'S_1'}=r(C_1)=\sqrt{5}$.

∴ $M=\overline{AR}\times(\overline{AR}+\overline{RO'}+\overline{O'S_1'})=3\sqrt{2}\times(4\sqrt{2}+\sqrt{5})$

$m=\overline{AR}\times(\overline{AR}-(\overline{O'S_2'}-\overline{O'R}))=3\sqrt{2}\times(4\sqrt{2}-\sqrt{5})$

∴ $Mm=18\times(32-5)$

$=27\times18=27\times(20-2)$

$=540-54=486$