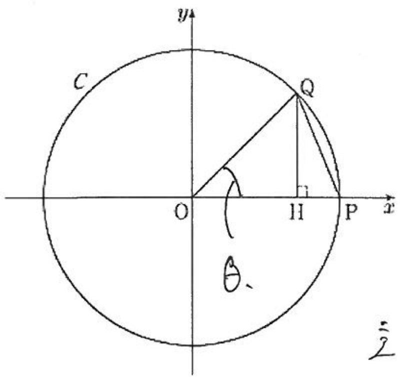
* 2019학년도 평가원 9월 수학 가형 19번.



자연수 n, 점 $P(2^n, 0)$

$\therefore \overline{OQ} = \overline{OP} = r = 2^n$

호 PQ의 길이가 π, $\angle QOP = \theta$라 하면

호 PQ의 길이 $= \pi = r\theta$ 에서 $\theta = \frac{\pi}{2^n}$.

$\therefore$ 점 $Q\left(2^n \times \cos\left(\frac{\pi}{2^n}\right), 2^n \times \sin\left(\frac{\pi}{2^n}\right)\right)$, 점 $H\left(2^n \times \cos\left(\frac{\pi}{2^n}\right), 0\right)$

$\overline{HP} = \overline{OP} - \overline{OH} = 2^n - 2^n \times \cos\left(\frac{\pi}{2^n}\right)$

$$\therefore \lim_{n\to\infty}\left(\overline{OQ} \times \overline{HP}\right) = \lim_{n\to\infty} 2^n \times 2^n \times \left(1 - \cos\left(\frac{\pi}{2^n}\right)\right) = \lim_{n\to\infty} \frac{2^{2n} \times \sin^2\left(\frac{\pi}{2^n}\right)}{1 + \cos\left(\frac{\pi}{2^n}\right)}$$

$$= \lim_{n\to\infty} \frac{\cancel{2^{2n}}}{1 + \cos\left(\frac{\pi}{2^n}\right)} \times \frac{\sin^2\left(\frac{\pi}{2^n}\right)}{\frac{\pi^2}{2^{2n}}} \times \frac{\pi^2}{\cancel{2^{2n}}} = \frac{\pi^2}{2}$$