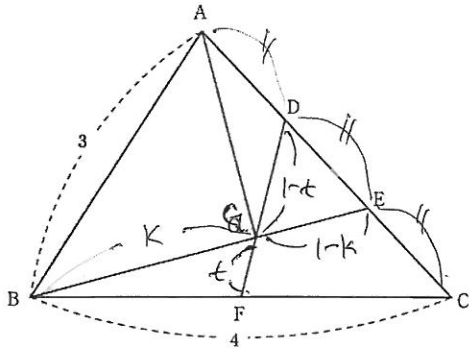


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$|\vec{AB}|, |\vec{BC}|$ 가 주어졌으므로 $\vec{AG} \cdot \vec{BE} = 0$ 을 활용할 때,
나머지 벡터 표현들을 \vec{AB}, \vec{BC} 로 나타내는 게 유리하다.

(점 G는 선분 FD를 $t:1-t$, 선분 BE를 $k:1-k$ 로 내분한다고
 t, k 를 설정)

$$\vec{BF} = \frac{2}{3}\vec{BC} + \frac{1}{3}\vec{BA}, \quad \vec{BD} = \frac{1}{3}\vec{BC} + \frac{2}{3}\vec{BA}, \quad \vec{FD} = \vec{FB} + \vec{BD} = -\frac{1}{2}\vec{BC} + \frac{1}{3}\vec{BC} + \frac{2}{3}\vec{BA} = -\frac{1}{6}\vec{BC} + \frac{2}{3}\vec{BA}$$

$$\left. \begin{aligned} \vec{FG} &= t\vec{FD} \quad (0 < t < 1) = -\frac{1}{6}t\vec{BC} + \frac{2}{3}t\vec{BA} \\ \vec{BG} &= k\vec{BE} \quad (0 < k < 1) = \frac{2}{3}k\vec{BC} + \frac{1}{3}k\vec{BA} \end{aligned} \right\} \begin{aligned} \vec{BG} &= \vec{BF} + \vec{FG} \\ \frac{2}{3}k\vec{BC} + \frac{1}{3}k\vec{BA} &= \frac{1}{2}\vec{BC} - \frac{1}{6}t\vec{BC} + \frac{2}{3}t\vec{BA} \end{aligned}$$

$$\therefore \frac{2}{3}k = \frac{1}{2} - \frac{1}{6}t \text{ 에서 } 4k = 3 - t \quad \text{--- ①}, \quad \frac{1}{3}k = \frac{2}{3}t \text{ 에서 } k = 2t \quad \text{--- ②}$$

①, ②에서 $t = \frac{1}{3}, k = \frac{2}{3}$, \therefore 점 G는 선분 BE를 2:1로 내분한다.

$$\therefore \vec{AG} = \frac{1}{3}\vec{AB} + \frac{2}{3}\vec{AE} = \frac{1}{3}\vec{AB} + \frac{4}{9}\vec{AC} = \frac{7}{9}\vec{AB} + \frac{4}{9}\vec{BC}, \quad \vec{BE} = -\frac{1}{3}\vec{AB} + \frac{2}{3}\vec{BC}$$

$$\vec{AG} \cdot \vec{BE} = -\frac{7}{27}|\vec{AB}|^2 + \frac{10}{27}\vec{AB} \cdot \vec{BC} + \frac{8}{27}|\vec{BC}|^2 = -\frac{63}{27} + \frac{10}{27}\vec{AB} \cdot \vec{BC} + \frac{128}{27} = 0$$

$$\therefore \vec{AB} \cdot \vec{BC} = -\frac{65}{10} = -\frac{13}{2}$$

$$|\vec{AC}|^2 = 9 + 16 - 2 \cdot 3 \cdot 4 \cdot \cos(\angle ABC) = (|\vec{AB} + \vec{BC}|)^2 = 9 + 16 + 2 \cdot \vec{AB} \cdot \vec{BC}$$

$$\therefore -24 \cdot \cos(\angle ABC) = -13 \text{ 에서 } \cos(\angle ABC) = \frac{13}{24} //$$

$$* \vec{AB} \cdot \vec{BC} = -\frac{13}{2} \Rightarrow |\vec{AB}| \cdot |\vec{BC}| \cdot \cos(\angle ABC) = 12 \cos(\angle ABC)$$

$$\therefore \cos(\angle ABC) = \frac{13}{24} // \quad (\because \vec{AB} \cdot \vec{BC} = -\frac{13}{2} \text{ 이면 } \vec{BA} \cdot \vec{BC} = \frac{13}{2} \text{ 이다})$$