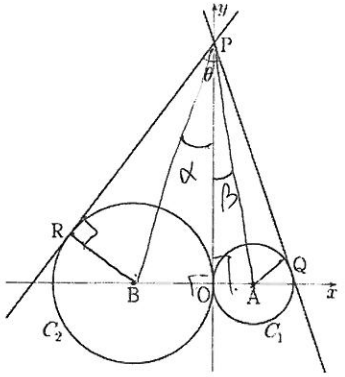
* 2019년 4월 시행 교육청 고3 수학 가형 29번.



$\angle BPO = \alpha$, $\angle APO = \beta$.

$\alpha + \beta = \frac{\theta}{2}$.

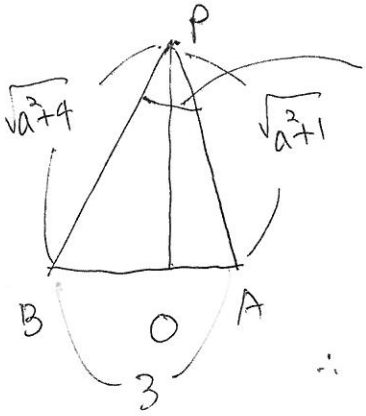
$\tan\theta = \frac{4}{3}$. $\tan\frac{\theta}{2} = t$ 라 하면 $\frac{2t}{1-t^2} = \frac{4}{3}$ 에서

$4t^2+6t-4 = (4t-2)(t+2) = 0$ 에서 $t = \frac{1}{2}$. ($\because 0 < \frac{\theta}{2} < \frac{\pi}{4}$).

$\therefore \tan\alpha = \frac{2}{a}$, $\tan\beta = \frac{1}{a}$. $\tan(\alpha+\beta) = \frac{\frac{3}{a}}{1-\frac{2}{a^2}} = \tan\frac{\theta}{2} = t = \frac{1}{2}$.

$\frac{\frac{3}{1}}{\frac{a^2-2}{a}} = \frac{1}{2}$. $\therefore \frac{a^2-2}{a} = 6$. $\therefore a^2-6a-2 = 0$. (덧셈정리)

따라서 $(a-3)^2 = a^2-6a+9 = 11$ ($\because a^2-6a = 2$) // (제2코사인).



$\tan\theta = \frac{4}{3}$ $\therefore 1+\tan^2\theta = \frac{25}{9} = \frac{1}{\cos^2\theta}$ 에서 $\cos\theta = \frac{3}{5}$.

$\cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2} = \frac{4}{5}$. $\therefore \cos\frac{\theta}{2} = \frac{2}{\sqrt{5}}$ ($\because 0 < \frac{\theta}{2} < \frac{\pi}{4}$).

$\therefore 9 = a^2+4+a^2+1-2\cdot\sqrt{a^2+4}\sqrt{a^2+1}\cdot\frac{2}{\sqrt{5}}$.

$\therefore \frac{16}{5}(a^2+4)(a^2+1) = (2a^2-4)^2 = 4(a^2-2)^2$ $\therefore \frac{4}{5}(a^4+5a^2+4) = a^4-4a^2+4$

$\therefore \frac{a^4}{5}-8a^2+\frac{4}{5} = \frac{1}{5}(a^4-40a^2+4) = 0$. $\therefore a^2 = \frac{40\pm\sqrt{40^2-4^2}}{2} = \frac{40\pm\sqrt{36\cdot 44}}{2}$

$\therefore a^2 = 20+6\sqrt{11}$ ($a^2 \neq 20-6\sqrt{11}$ 증명은 생략; $a > \sqrt{2}$). $\therefore a = \sqrt{20+2\sqrt{9\cdot 11}} = 3+\sqrt{11}$.

$\therefore a^2-6a+9 = (a-3)^2 = (\sqrt{11})^2 = 11$ //