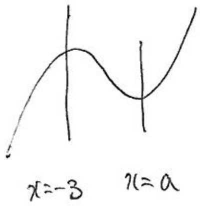
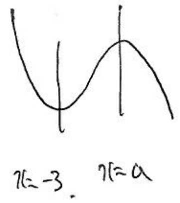
* 2019년 7월 시행 교육청 고3수학 나형 30번.

$f(x)$의 개형

(i)

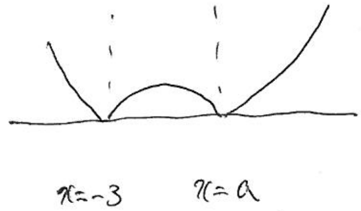


(ii)



$|f'(x)|$의 개형은 (i) (ii) 모두

$\Rightarrow$



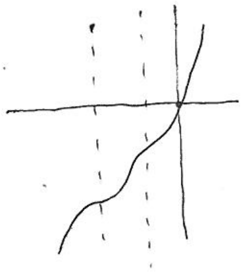
$$g(x)=\begin{cases} f(x) & (x<-3) \\ \int_0^x |f'(t)|\,dt & (x\geq -3) \end{cases}$$

$\rightarrow$ $(-3, 0)$ 구간에서 $g(x)<0$, $\because$ 역방향 적분.

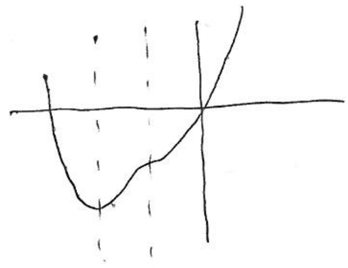
(가) $g(-3)=-16$, $g(a)=-8$ (나) $g(x)$는 연속, (다) $g(x)$는 극솟값 존재

$\int_0^a |f'(t)|\,dt = g(a) = -8$ 이므로 $-3<a<0$, $g(0)=0$.

$g(x)$의 개형. (i) $f(x)-$(i)



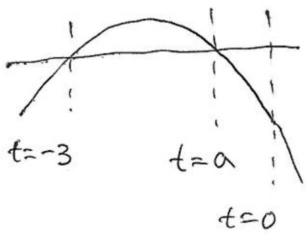
(ii) $f(x)-$(ii)



$\therefore$ (ii) 개형이어야 한다.

($\because$ $g(x)$는 극솟값 존재).

따라서 $f'(t)$의 개형은



$\rightarrow$ $f'(t)=k(t+3)(t-a)$

$=k(t^2+(3-a)t-3a)$

$\therefore \int_0^a f'(t)\,dt = 8$. $\int_0^{-3} f'(t)\,dt = 0$.

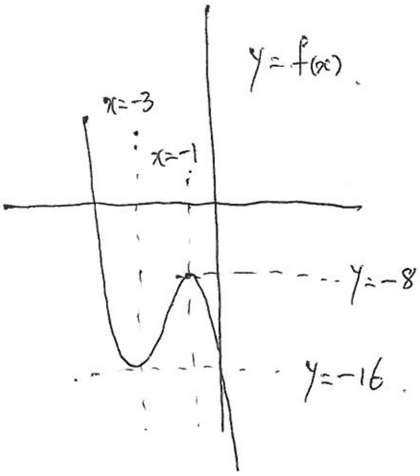
$$\int_0^{-3} f'(t)\,dt = k\left[\frac{t^3}{3}+\frac{(3-a)}{2}t^2-3at\right]_0^{-3} = k\times\left(-9+\frac{27}{2}-\frac{9}{2}a+9a\right)=k\left(\frac{9}{2}+\frac{9}{2}a\right)=0.$$

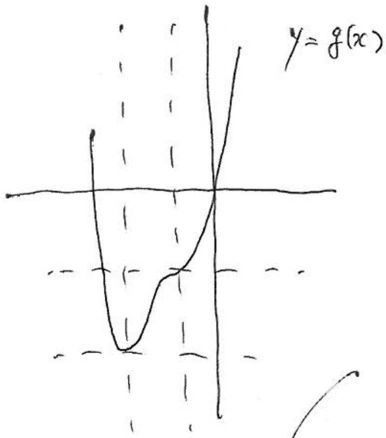
$$\int_0^a = \int_0^{-1} = k\times\left(-\frac{1}{3}+2-3\right) = -\frac{4}{3}k = 8 \qquad \therefore a=-1,\ k=-6.$$

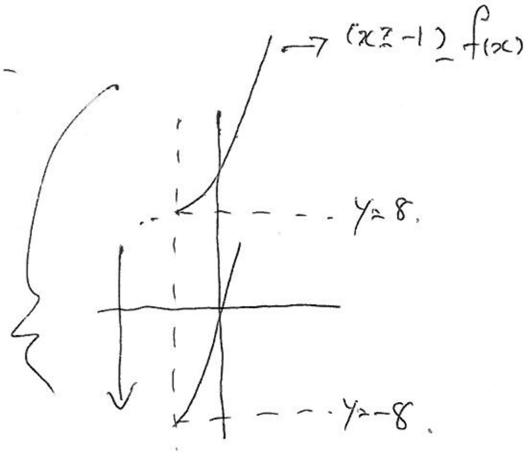
$$\therefore f'(x) = -6(x^2+4x+3) = -6x^2-24x-18.$$

$$f(x) = -2x^3-12x^2-18x-16 \quad (\because f(-3)=g(-3)=-16).$$

따라서 $f(x)$와 $g(x)$의 그래프를 나타내면 다음과 같다.







$$\therefore\ g(x) = \begin{cases} f(x) & (x < -1) \\ -f(x)-16 & (x \geq -1) \end{cases}$$

따라서 $\left| \int_a^4 \{f(x)+g(x)\}\,dx \right|$

$$= \left| \int_{-1}^4 \{f(x) - f(x) - 16\}\,dx \right| = \left| \int_{-1}^4 -16\,dx \right| = 16 \times 5 = 80$$

($\because$ 적분구간이 $[-1, 4]$이므로 $(x \geq -1)$에서 $g(x) = -f(x)-16$이 성립)