6-1. Find the exact arc length of the curve $x=\frac{1}{8}y^4+\frac{1}{4}y^{-2}$ over the interval from $y=1$ to $y=4$.

6-2. Find the length of the curve $y=\ln\left(\frac{e^x+1}{e^x-1}\right)$, $a\le x\le b$, $a>0$.

6-3. Use Simpson's Rule with $n=10$ to estimate the arc length of the curve $x=y+\sqrt{y}$, $1\le y\le 2$. Compare your answer with the value of the integral produced by your calculator.

6-4.

(a) The figure (6-4) shows a telephone wire hanging between two poles at $x=-b$ and $x=b$. It takes the shape of a catenary with equation $y=c+a\cosh\left(\frac{x}{a}\right)$. Find the length of the wire.

(b) Suppose two telephone poles are 20m apart and the length of the wire between the poles is 20.4m. If the lowest point of the wire must be 9m above the ground, how high up on each pole should the wire be attached?

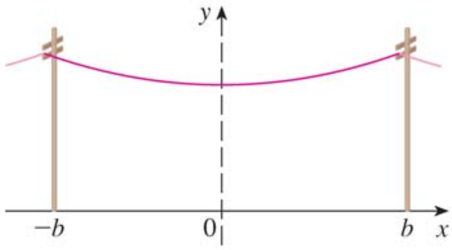

figure (6-4)

[Hint : $g(x)=x\sinh\left(\frac{10}{x}\right)-10.2 \Rightarrow g(28.95)=0$. You don't need to evaluate the exact value i.e., leave the $\cosh$ value as it is.]

6-5. Find the exact area of the surface generated by revolving the curve $8xy^2=2y^6+1$, $1\le y\le 2$ about the y-axis.

6-6. Figure (6-6) shows a spherical cap of height h cut from a sphere of radius r.

(a) Show that the surface area S of the cap is $S=2\pi rh$.

(b) The portion of sphere that is cut by two parallel planes is called a <u>zone</u>. Use part (a) to show that the surface area of a zone depends on the radius of the sphere and the distance between the planes, but not on the location of the zone.

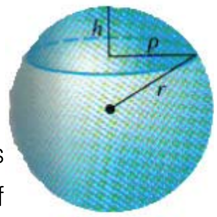

figure (6-6)

6-7. If a cone of slant height l and base radius r is cut along a lateral edge and laid flat, then as shown in figure (6-7) it becomes a sector of a circle of radius l.

(a) Use the formula $A=\frac{1}{2}l^2\theta$ for the area of a sector with radius l and central angle θ (in radians) to show that the lateral surface area of the cone is πrl.

(b) Use part (a) to obtain a formula $S=\pi(r_1+r_2)l$ for the lateral surface area of a frustum.

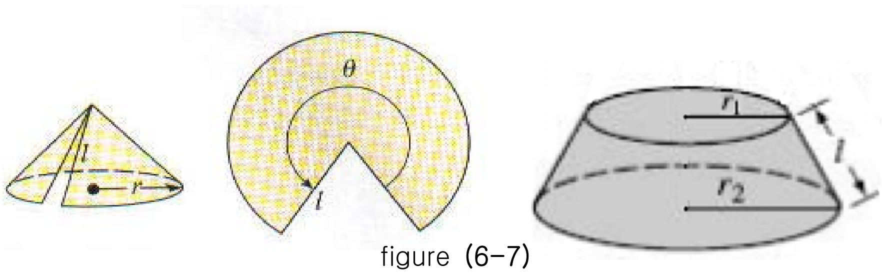

figure (6-7)

6-8. Let $y=f(x)$ be a smooth curve on the interval $[a,\ b]$ and assume that $f(x)\geq 0$ for $a\leq x\leq b$. By the Extreme Value Theorem, the function f has a maximum value K and a minimum value k on $[a,\ b]$.

Prove the following :

If L is the arc length of the curve $y=f(x)$ between $x=a$ and $x=b$ and if S is the area of the surface that is generated by revolving this curve about the x-axis, then $2\pi kL\leq S\leq 2\pi KL$.

6-9. If the region $\Re=\left\{(x,\ y)\,|\,x\geq 1, 0\leq y\leq \dfrac{1}{x}\right\}$ is rotated about the x-axis, the volume of the resulting solid is finite. Show that the surface area is infinite. (The surface is shown in figure (6-9) and is known as Gabriel's horn.)

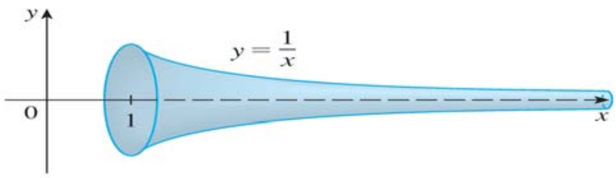

figure (6-9)

6-10.

(a) If $a>0$, find the area of the surface generated by rotating the loop of the curve $3ay^2=x(a-x)^2$ about the x-axis.

(b) Find the surface area if the loop is rotated about the y-axis.

6-11.

(a) The ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, $a>b$ is rotated about the x-axis to form a surface called an <u>ellipsoid, or prolate spheroid</u>. Find the surface area of this ellipsoid.

(b) If the ellipse in part (a) is rotated about its minor axis (the y-axis), the resulting ellipsoid is called an <u>oblate spheroid</u>. Find the surface area of this ellipsoid.

6-12.

(a) Show that an observer at height H above the north pole of a sphere of radius r can see a part of the sphere that has area $\dfrac{2\pi r^2 H}{r+H}$.

(b) Two spheres with radii r and R are placed so that the distance between their centers is d, where $d>r+R$. Where should a light be placed on the line joining the centers of the spheres in order to illuminate the largest total surface?

6-13. The figure (6-13) shows a semicircle with radius 1, horizontal diameter PQ, and tangent line at P and Q. At what height above the diameter should the horizontal line be placed so as to minimize the shaded area?

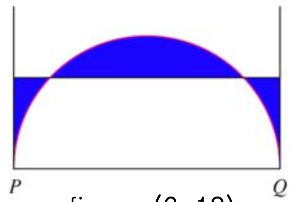

figure (6-13)

Recommended Problems :

Section 8.1 : 11, 15, 31, 32, 38, 39, 41

Section 8.2 : 10, 12, 16, 20, 26, 35, 36